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Impact of the formation and ablation of Antarctic ice sheet on the global geoid and sea level

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Abstract It is convenient to investigate the gravimetry using a harmonic spheric function for the description of the distribution and thickness of the Antarctic ice sheet. The gravitational theory and the Stokes' harmonic spheric function formula were used to determine the impact of the Antarctic ice cap on the global geoid. The Antarctic ice cap is formed from the condensation of seawater vapour whose mass is equal to a layer of seawater 59 m thick of covering the earth's surface, i. e. 2.7×10^{19} kg. This will cause the global averaged geoid to decrease for around 23 m. The authors' computations show that the Antractic ice cap has a great impact on the global geoid, which increases (+) in some regions, but decreases (−) in other reigions. The geoid is +115 m, −37 m and +8 m at the South Pole, the 25°S parallel and the North Pole, respectively. If the Antarctic ice cap melts completely, on the rigid Earth's surface the seawater and geoid will return to its original position (and height) due to the balancing force of the fluid. Since the crust is almost in a state of isostasy, assuming that the crust is an elastic solid and the mantle is an incompressible fluid, the load of seawater will deflect the crust and drive the mantle material to flow. The material above the isostatic surface compensates mutually. If the densities of the mantle and seawater are 3270 kg/m^3 and 1030 kg/m^3, respectively, then the variation in the elevation of the continent is only 2.8 m with respect to the sea level after the Antarctic ice cap melts;it is not larger than that estimated by some people. It is worth noting that the above results were derived from an ideal Earth model. In the real Earth, the mantle and crust are visco-elastic.

Key words Antarctic ice sheet, geoid undulation, relative change of sea level.

1 Introduction

The Antarctic ice sheet nearly occupies 90% of the global ones, the formation and ablation of which have a strong impact on the global geoid and the relative sea level. This problem attracts the attention of geodesists as well as oceanographers, weathermen, environmentalists and catastrophers in late thirty years. Some valuable works are completed by Kivioja (1967) and Bretterbaner (1990) and others. Bretterbaner indicates that the problem seems to be related to the enormous mass migration. Based on the predecessors' works, the point mass distribution in the paper of Kivioja (1967) is here substituted by the space distribution of ice sheet, thus the determination of the geoid variation is more practicable. At the same time, the relative change of global sea level is estimated by using the idea that the fluid media in solid crust and under crust are in a state of static equilibrium on the compensation surface, the obtained result is reasonable more than that in the paper of Kivioja (1967) and this result is, smaller than that estimated by some authors.

2 Antarctic ice thickness and its spheric harmonic expression

In some documents it is reported that the most parts of sea level in the Antarctic Continent are covered by the ice body, the rocks only make up less than 1/10 of the whole areas (Dubach and Tabern 1972; Zhang 1988). For the sake of the description of the paper, it is supposed that the areas are covered by ice sheets on the sea level. According to the topographic elevation given by Seyawa (1980) (see Fig. 1a), it can be known that the distribution of ice sheets is closely symmetrial about polar point, and that the highest elevation is not far away from polar point, forming the relief high centrally and low around. For convenience, the height $H(\theta,\lambda)$ of ice sheet is fitted by the spheric function, where θ is colatitude and λ longitude. Obviously, the expanded form of spheric function is:

$$H(\theta,\lambda) = \sum_{M=2}^{N} H_n(\theta,\lambda) = \sum_{n=2}^{N}\sum_{M=2}^{N}(A_{nm}\cos m\lambda + B_{nm}\sin m\lambda)P_{nm}(\cos\theta)$$

$$A_{nm} = \frac{(n-m)!(2n+1)}{(n+m)!4\pi}\int_0^{\pi}\int_0^{2\pi} H(\theta,\lambda)\cos m\lambda P_{nm}(\cos\theta)\cos\theta \mathrm{d}\theta \mathrm{d}\lambda$$

$$B_{nm} = \frac{(n-m)!(2n+1)}{(n+m)!4\pi}\int_0^{\pi}\int_0^{2\pi} H(\theta,\lambda)\sin m\lambda P_{nm}(\cos\theta)\cos\theta \mathrm{d}\theta \mathrm{d}\lambda$$

where A_{nm} and B_{nm} are coefficients of relief height of n degrees and m orders, $P_{nm}(\cos\theta)$ is associated Legendre's function. Above formulas are spread to 180 degrees, obtaining the Fig. 1b, which has the difference of about 40 m from the original data (Fig. 1a).

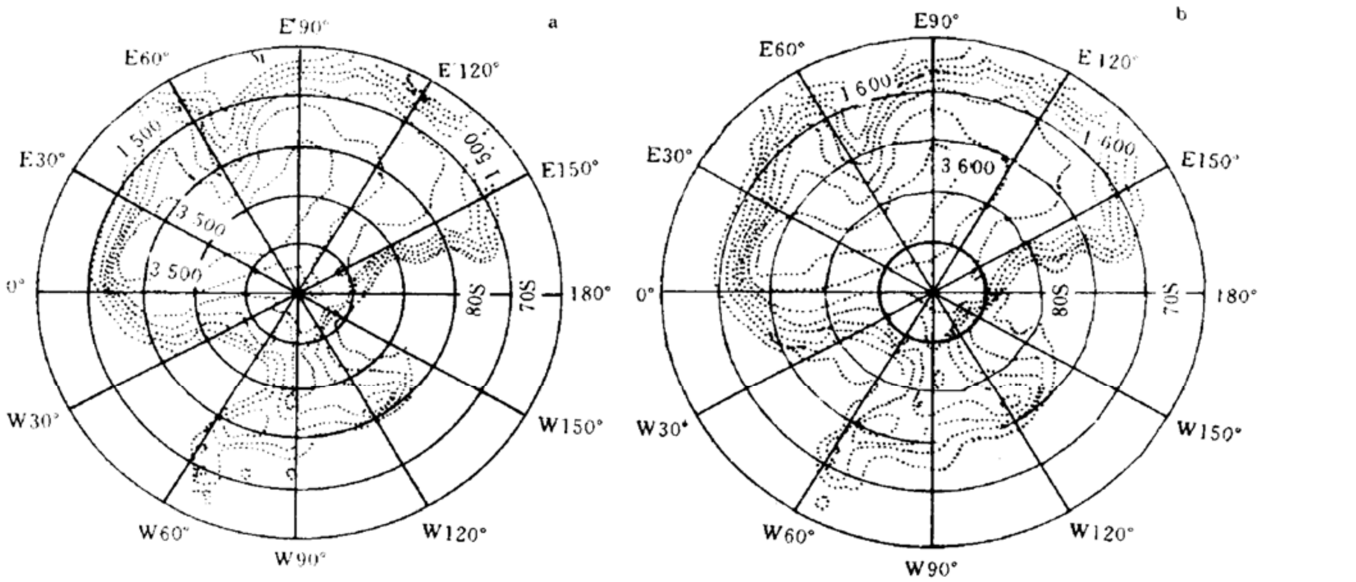

Fig. 1. Antarctic topography (a) and Antarctic topography shown by spheric function (b)(unit: m).

3 Influence of ice sheet on geoid

It is assumed that the ice sheet with thickness $H(\theta,\lambda)$ is compressed into the bedding surface, and the surface density is $\delta_i H$, where δ_i is density of ice body. The gravity effect δg_H, caused by which, can be closely expressed by formula: $\delta g_H = 2\pi G \delta_i H$, $\delta_i = 1030$ kg/m^3, where G is gravitation constant.

Based on the Stokes formula on the spheric surface, the effect δN of ice sheet on geoid can be obtained: $\delta N = \frac{R}{4\pi r}\int_0^{\pi}\int_0^{2\pi}\delta g_H S(\Psi)\,\mathrm{d}\sigma$, where d$\sigma$ is surface element. The Stokes function: $S(\Psi) = \sum_{n=2}^{N}\frac{2n+1}{n-1}P_n(\cos\Psi)$, where $P_n(\cos\Psi)$ is the Legendre polynomials, where Ψ is angular separation of spheric surface, R is radius of the Earth, r is normal gravity value. According to the definitions of addition and orthogonality of spheric function, above formula can be reduced to: $\delta_N = 0.0199\sum_{n=2}^{N}\frac{1}{n-1}H_n$, where H_n is spheric function of n degrees, its expression can be obtained by former formula, in which n is taken up to 180, and the unit of H_n is meter. The results of δg_H and δN refer to Fig. 2a,b. Therefore, it could be seen that the effect of ice sheet, after being formed, on geoid is mainly in the southern hemisphere, and the most effect is in the South Pole, in which the δN value is up to 138 m, and δg up to 119×10^{-5} m/s^2. The biggest gravity effect arises on these regions, because of the most concentrated mass. The smallest value of δg and δN is nearly in 45°S and 25°S, and them reaches 5.7×10^{-5} m/s^2 and 14.8 m respectively in the North Pole.

The integral mass of Antarctic ice sheet is 2.7×10^{19} kg, it is constituted by evaporiting and condensating the sea water of more than 70% of the worldwide areas, which corresponds to the sea water mass of 59 m thickness in the ocean, and makes the global geoid to cut down 23 m on the average. Thus, the geoid variation in ocean region could be estimated roughly by $\Delta N = \delta N - 23$ m (see Fig. 3), it is changed with the different latitude. The geoid distributed between 65° - 90°S is uplifted, attaining the biggest value of 115 m near the South Pole, but the geoid in other parallels of latitude, including the South Pole, is obviously lowered, and the smallest value -37 m is in 25° parallel of southern latitude. These are the variations of global geoid after the formation of Antarctic ice sheet. As for the change after ablation, the whole world would equally increase 23 m. It is worth notice that, in above discussion, the earth is regarded as a rigid body, only considering the effect of gravitational force caused by the circulation of ice and water on earth, but not considering the effect of the water formation due to the mantle gas-separation and the volumetric variation of oceanic basin due to the construction and others.

4 Relative sea level change after ablation of ice sheet under condition of isostasy

Assuming that the solid crust presents the elasticity, and the mantle under crust presents the incompressible fluid state, from the isostatic principle it can be known that the pressure intensity in centain depth of mantle should be equal everywhere. If a layer of sea water, in which the thickness and density are Δd (m) and ρ_w (kg/m^3) respectively, is added to the oceanic surface, the solid crust would be bended downward because of the elasticity, but its thickness under loading is not changed. At present, the mantle in asthenosphere starts flowing and pressing towards all round,

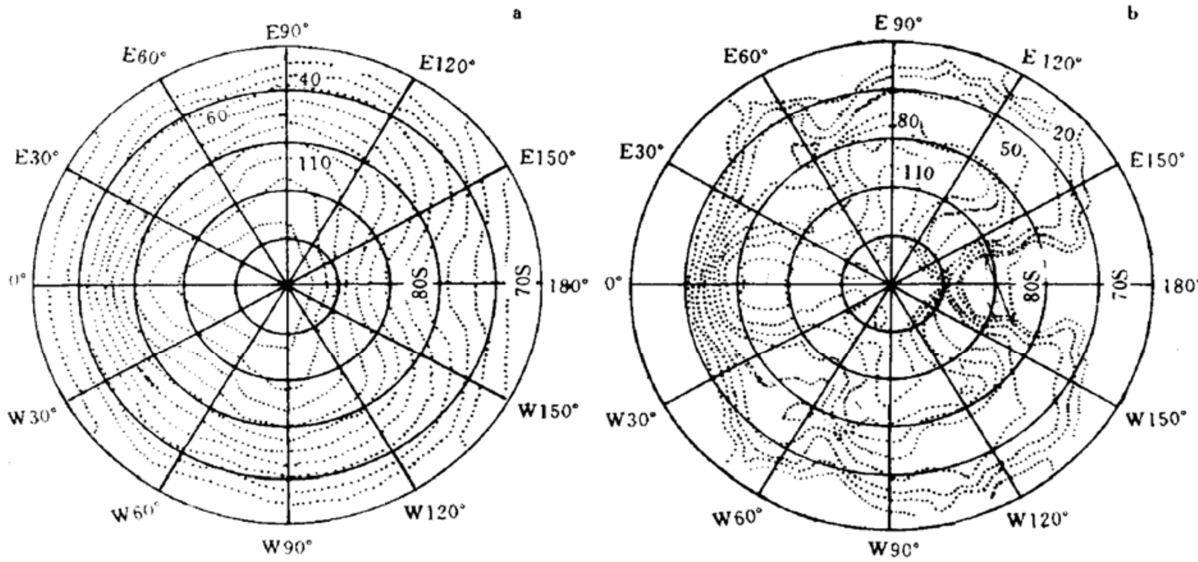

Fig. 2. Effect of the Antarctic ice sheet on the geoid (in m) (a) and on the gravity (in 10^{-5} m/s^2)(b).

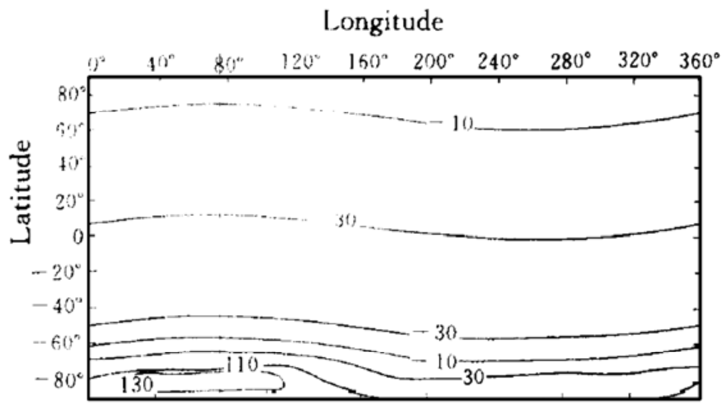

Fig. 3. Influence of the Antarctic ice sheet on the geoid.

and the thickness of mantle would be thinned under the isostatic adjustment. In fact, the elasticity and viscoelasticity of earth obviously exist in the tide observation of solid earth.

Because of the large scale global isostatic adjustment, the thickness ΔD (m) of mantle body would be changed with the thickness Δd (m) of sea water which covered on the mantle body. Now $\Delta D = \Delta d\rho_w/\rho_m$, in which the $\Delta d = 65$ m (including the ablation of the ice sheet in Greenland, etc.), $\rho_w = 1030$ kg/m^3 and different mantle densities ρ_m (kg/m^3) are substituted, then:

$\rho_m = 3270$ kg/m^3, $\Delta D = 20.5$ m;
$\rho_m = 3370$ kg/m^3, $\Delta D = 19.9$ m;
$\rho_m = 3470$ kg/m^3, $\Delta D = 19.3$ m.

In the process of thinning of oceanic mantle, the plastic flow body enters under the continental crust, leading the crust to rise. The rise value is ΔH, $\Delta H = 70/30\Delta D$, where the unit of ΔH and ΔD is meter, 70/30 is the ratio between the ocean and land area. For:

$\rho_m = 3270$ kg/m^3, $\Delta H = 47.8$ m;
$\rho_m = 3370$ kg/m^3, $\Delta H = 46.4$ m;
$\rho_m = 3470$ kg/m^3, $\Delta H = 45.0$ m.

If the relative change of sea surface is Δh, $\Delta h = 65 - (\Delta D + \Delta H)$, the unit of Δh is m also. when:

$\rho_m = 3270$ kg/m^3, $\Delta H = 2.8$ m;
$\rho_m = 3370$ kg/m^3, $\Delta H = 1.3$ m;
$\rho_m = 3470$ kg/m^3, $\Delta H = -0.7$ m.

From above values it could be known that the relative change of global sea level is smaller, which is related with the mantle density. When the mantle density is 3270 kg/m^3, the relative rise of sea level is only 2. 8 m.

5 Conclusions

(1) The influence of the Antarctic ice sheet (constituted by migration of global materials) on the geoid is considerable, showing that the big influence is in two poles, the small one in the middle and the largest one in South Pole in the world. It is convenient to compute the influence using the spheric function, and the computational results are more practicable.

(2) The crust is basically on the condition of isostatic balance, namely, the load of sea water could be balanced by the compensated flowage of materials under the deep crust. When the sea water in thickness of 65 m is added on the ocean crust, the relative variation of sea level between land and ocean is only 2. 8 m, but the interval of isostatic adjustment achieved by crust could be considerably long, because the practical mantle has the viscoelasticity.

(3) The formation of Antarctic ice sheet passes through a long historical period, therefore the ablation of ice sheet passes through a long time also. The earth under the action of a long-term load (and unload) force could develop the creep, leading to the postglacial rebound and the change of geoid and relative sea level. These problems are studied by some scholars using the strict rheological theory (Hang 1994) in recent years. Providing that the viscoelasticity of earth and the history of ice sheet formation (ablation) are understood comprehensively, the study could be practicable, but the pity is that the knowledge about it is not many. Comparing with above objective, the problems are only primarily discussed by using the condition of ideal earth in this paper, and hereafter should be studied detailedly.

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