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Determination of Antarctic geoid by using global gravity field

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Abstract With Chinese latest global gravity field model WDM94, the authors provide the geoid height and mean free-air gravity anomaly of Antarctica (The range of latitude is from - 60° to - 90°). In order to conclude and analyze the characters of Antarctic geoid roundly, the authors collect the latest oversea global gravity field model OSU91 (to degree and order 360) and JGMOSU (to degree and order 360), get the corresponding geoid height and mean free-air gravity anomaly. The results are compared with the results got from WDM94, thus we get the difference. The standard deviation of geoid height between WDM94 and OSU91 is ±1.90 m; the deviation of geoid between WDM94 and JGMOSU is ±2.09 m. The standard deviation of mean gravity anomaly are ±8.97 mGal and ±9.32 mGal respectively.
Key words Antarctica, geoid, gravity anomaly, gravity field model.

1 Introduction

Gravity field is the most important physical character of the Earth. It has restrict relation to all physical events happening on the planet or in the near space. Gravity is most universal property of all the matters in the universe, it restricts the change and development of the universe. The research of gravity field change is an important content in studying of global changes. Antarctic gravity field restricts the change of Antarctic icecap, sea level, climate and global geoid. It reflects the character and distributing of matter within the Earth, and also the relation between the burden of ground ice and snow and the structure of the Earth's crust. On the other hand, the changes of Antarctic icecap, sea level and climate affect directly all kinds of spatial and physical character of the environment of Antarctic ice and sea, these changes also affect gravity field of the Earth.

Because the weather is very cold and the nature condition is very hard and ice and snow cover the great part of area, it is very hard to carry out gravity survey in Antarctic. In this region some countries have carried out some small-scale and small-area gravity survey. Because of the lack of large area gravity data, we have met with much difficulty in the research of the gravity field of Southern Hemisphere.

Since the first man-made Earth satellite was launched, Keefe concluded from the satellite orbit perturbation data that global geoid is the pyriform. It is convex in North Pole and concave in South Pole. With the development of satellite hypsometry, the data we owned are from five satellites, whose names are GEOS-3, SEASAT, GEOSAT, ERS-1 and TOPEX/DOSEIDON. The spatial definition of these satellites is ameliorating better and better and the precision higher and higher. These data, which are of great importance to research of Antarctic gravity field, help people understand the ocean gravity field better. Among these data, the data from GEOSAT/GM is of necessary veracity and repetitiousness using it in reflecting the geotectonic detail of global ocean gravity field and in surveying ocean geoid with definition of 10 km(Chen *et al*. 1995). Because of strategic and martial meaning, American government had opened the data from south of 60°S in 1990, then data from south of 30°S in 1992. The data of Satellite Altimetry played an important role in research on the geoid of Antarctic Ocean. The ocean geoid got from the fore-unopened data on the south of 60°S solves some opened important problems of oceanography. For example, to determine the bypast velocity of some blocks, the growth and shape of ocean ridge and some other benthal structure character. Opened GEOSAT/GM data provide density data circle around Antarctica, with which we could reckon high-definition (about 10 km) and high-accuracy (4 ~ 5 mGal) global gravity field (Li *et al*. 1996).

Since a large part of area of Antarctic is covered with ocean-ice, large-scale gravity survey can´t be carried out and satellite data should be used mainly. No data of satellite altimetry can be used in Antarctic glacier, snow-cap and land, so we can only analyze Antarctic geoid with data of satellite orbit perturbation. Since these data can´t be obtained, it´s reasonable for us to use high-degree global gravity field model. The low-degree part of high-degree gravity field model includes the information of satellite orbit perturbation, while the high-degree part includes the information of satellite altimetry data.

This paper adopts three global gravity field models: two of which (OSU91 and JGMOSU) are the latest and the highest-precision gravity field model to degree and order 360, the third model WDM94 to degree and order 360 was developed by China(Ning *et al*. 1994).

2 Principle and method

Global gravity field model is expressed with sphere-harmonic expansion. To ascertain the sphere-harmonic coefficient, the observation value of Earth gravity field, as is observation value of gravity, should be known. In fact, it´s impossible to get the observation value all over the Earth. Usually grid-average gravity values were instead of demanded world-wide and continuous gravity value theoretically, grid gravity values were used to calculate the global disturbance potential T, thus the gravity anomaly and geoid height were ascertained. The followings are the basic theory that the disturbance potential is expressed with sphere-harmonic series.

In point $P(r, \theta, \lambda)$, the sphere-harmonic expression of any Newton gravity potential V is:

$$V(r, \theta, \lambda) = \sum_{n=0}^{\infty} \left| \frac{a}{r} \right|^{n+1} \sum_{m=0}^{\infty} (\overline{C}_{nm} \cos m\lambda + \overline{S}_{nm} \sin m\lambda) \overline{P}_{nm}(\cos\theta) \quad (1)$$

From the above formula, supposing that there is no mass out of the Earth, we could get the sphere-harmonic expression of global gravity potential V as follows:

$$V(r, \theta, \lambda) = \frac{GM}{r} \left| 1 + \sum_{n=0}^{\infty} \left| \frac{a}{r} \right|^{n} \sum_{m=0}^{\infty} (\overline{C}_{nm} \cos m\lambda + \overline{S}_{nm} \sin m\lambda) \overline{P}_{nm}(\cos\theta) \right| \quad (2)$$

In above formula, r is the distance from the point to the Earth´s core, θ is the colatitude and λ is the longitude of the point, G is the gravity constant of the Earth´s core, M is the mass of the Earth, a is the semi-major axis of the reference ellipsoid, C_{nm} and S_{nm} is completely-normalized Earth potential coefficients, $\overline{P}_{nm}(\cos\theta)$ is completely-normalized first-class Legendre association function. The sphere-harmonic expanding formula of normal potential U of any point $P(r, \theta, \lambda)$ on Earth surface is:

$$U(r, \theta, \lambda) = \frac{GM}{r}\left[1 + \sum_{n=1}^{\infty}\left(\frac{a}{r}\right)^{2n}\overline{C}'_{2n}\overline{P}_{2n}(\cos\theta)\right] \tag{3}$$

In formula (3), since the rotary ellipsoid whose angle velocity equals with the Earth´s autorotation angle velocity ω is selected as the reference normal gravity field, the variable of longitude disappears; Since the normal gravity field is symmetrical about the equator surface, only even-number items of zonal spherical function exist while positive and negative odd-number items of it offset each other. Supposing that the mass of Earth equals with the mass of the reference ellipsoid and their cores of mass are coincided, the zero-order item and one-order item of the disturbance potential will disappear. So the sphere-harmonic expression of the disturbance potential T is:

$$\begin{aligned} T(r, \theta, \lambda) &= V(r, \theta, \lambda) - U(r, \theta, \lambda) \\ &= \frac{GM}{r}\sum_{n=2}^{\infty}\left(\frac{a}{r}\right)^{n}\sum_{m=0}^{\infty}(\overline{C}^{*}_{nm}\cos m\lambda + \overline{S}_{nm}\sin m\lambda)\overline{P}_{nm}(\cos\theta) \end{aligned} \tag{4}$$

In the above formula: $\overline{C}^{*}_{no} = \overline{C}_{no} - \overline{C}'_{no}$, $n = 2k, k = 1, 2, 3, \cdots$; $\overline{C}^{*}_{nm} = \overline{C}_{nm}$

Molodensky boundary value condition of sphere approximate is:

$$\Delta g(r, \theta, \lambda) = -\frac{\partial T(r, \theta, \lambda)}{\partial r} - \frac{2}{r}T(r, \theta, \lambda) \tag{5}$$

Supposing that the global is covered completely with average gravity anomaly, from formula (5) and formula (4), using the orthogonal relation of sphere-harmonic expanding, potential coefficients can be decided as follows:

$$\left.\begin{aligned} \overline{C}^{*}_{nm} &= \frac{1}{4\pi r(n-1)}\iint_{\sigma}\Delta g(\theta, \lambda)\overline{P}_{nm}(\cos\theta)\cos m\lambda \, d\sigma \\ \overline{S}_{nm} &= \frac{1}{4\pi r(n-1)}\iint_{\sigma}\Delta g(\theta, \lambda)\overline{P}_{nm}(\cos\theta)\sin m\lambda \, d\sigma \end{aligned}\right\} \tag{6}$$

The disturbance potential T, geoid height N and gravity anomaly Δg can be calculated with formula (4) by using potential coefficients got from formula (6).

The high-degree potential coefficients got from harmonic analysis of terrestrial data but the precision of the low-degree part is not high, because of that the distribution of the gravity data is not even and the average value is got every 30′. On the contrary, satellite moving in the space is sensitive not to the high-frequency information, but to the low-frequency information of the high altitude gravity field. Thus the low-degree part of gravity field could get through the data of tracing satellite. And the low-degree part of high-degree gravity field model is usually worked out through the combination of satellite gravity field model and terrestrial gravity data.

3 The value results

In this paper the $30' \times 30'$ grid average geoid height and air gravity anomaly were calculated at Antarctic area of Latitude -60° – -90° and Longitude 0° – 360°, which is based on WDM94 model, totally there are 43200 values. The value statistics results are listed in Table 1, and the figure of isolines is shown in Fig. 1 and Fig. 2.

Table 1. Geoid height and gravity anomaly statistics of WDM94

Sort	Average	Maximum	Minimum
Geoid height/m	− 17.9	44.3	− 69.0
Air gravity anomaly /mGal	− 7.849	328.6	− 307.25

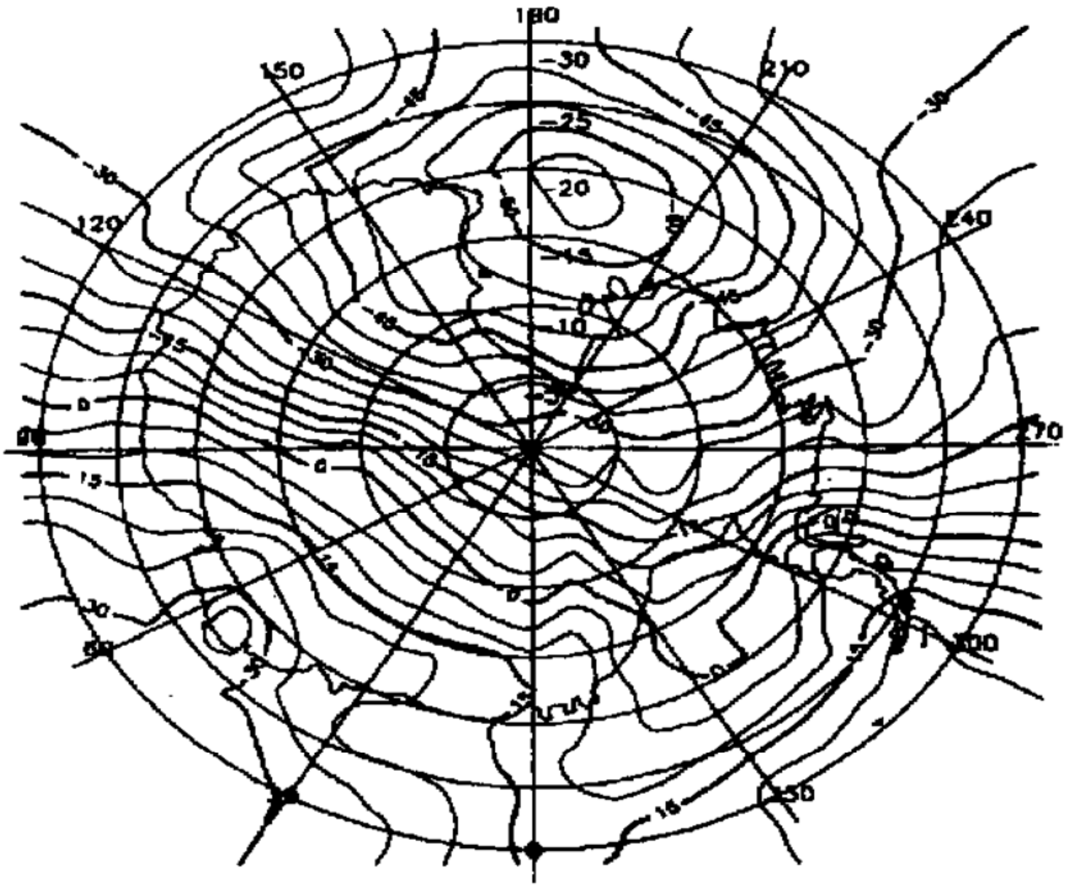

Fig. 1. Geoid height isolines of WDM94 model (unit: m, interval of isolines: 5 m).

From Fig. 1 Antarctic geoid value at the longitude area from 90° to 270° is negative, there is larger negative at the place near $B = -71^\circ$, $L = 190^\circ$, and the minimum negative is − 69.0 m, the pole is concave about 22 m from the reference ellipsoid. Antarctic geoid value at the longitude area from 270° to 90° is positive, the maximum is 44.3 m and the location is near $B = -53^\circ$, $L = 67^\circ$. At the area of longitude from 180° to 360° and latitude from -80° to -90° gravity anomaly changes rapidly and there are large negatives and large positives.

Considering that there is not accurate geoid can be taken as reference gravity field in Antarctica, the paper compares the results got from OSU91 model and JGMOSU model to those got from WDM94 model. The isolines figures are shown from Fig. 3 to Fig. 6,

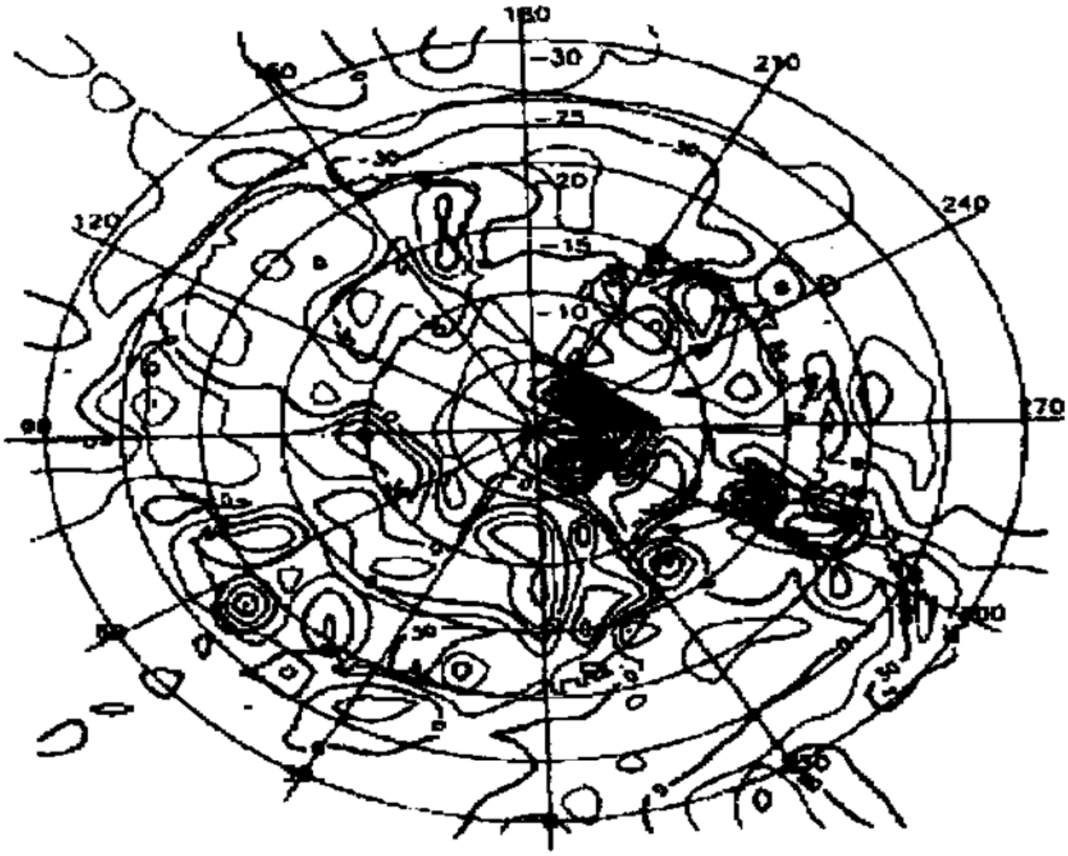

Fig. 2. Air gravity anomaly isolines of WDM94 model (unit: mGal, interval of isolines: 15 mGal)

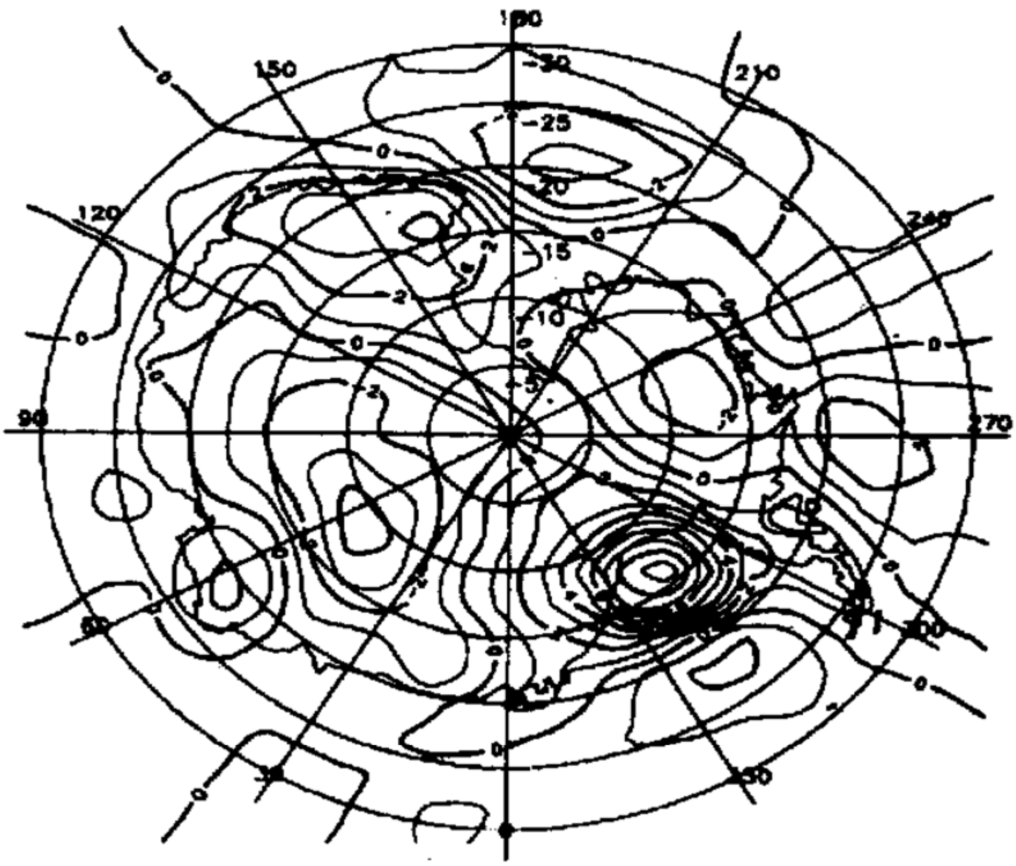

Fig. 3. The isolines of geoid differences between WDM94 model and OSU91 model (unit: m, interval of isolines: 1 m).

the compared results of statistics are shown in Table 2 and Table 3, the statistics figures are shown from Fig. 7 to Fig. 10.

Table 2. The statistics of geoid height and air gravity anomaly differences between WDM94 model and OSU91

Sort	Average	Maximum	Minimum	Standard deviation
Geoid height/m	− 0.07	10.00	− 4.90	±1.90
Air gravity anomaly/mGal	− 0.37	69.20	− 84.60	±8.97

Table 3. The statistics of geoid height and air gravity anomaly differences between WDM94 model and JGMOSU

Sort	Average	Maximum	Minimum	Standard deviation
Geoid height/m	0.06	7.20	− 6.00	±2.09
Air gravity anomaly/mGal	− 0.32	71.60	− 80.40	±9.32

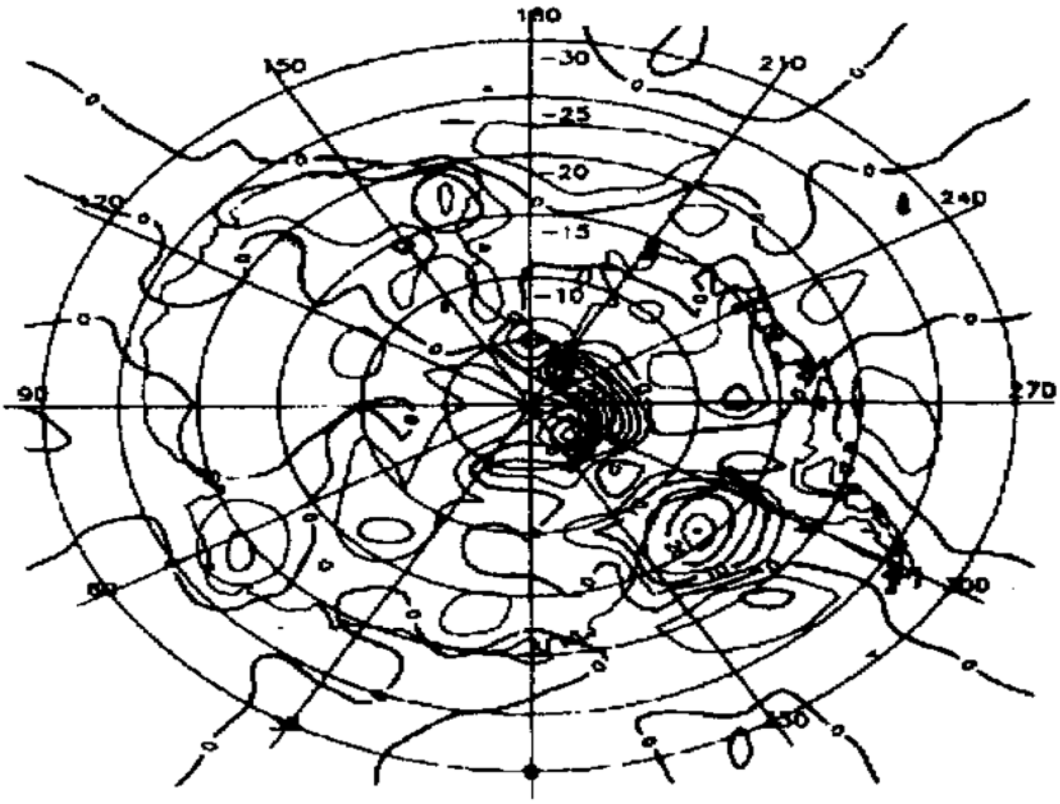

Fig. 4. The isolines of air gravity anomaly differences between WDM94 model and OSU91 model (unit: mGal, interval of isolines: 5 mGal).

From Fig. 4 (or Fig. 6), gravity anomaly of WDM94 and OSU91 (or JGMOSU) have great difference near the pole, the maximum positives and the minimum negatives are also distributed in this area.

4 Summary

Because of the lack of Antarctic field-survey gravity data, global gravity field models were adopted in research of Antarctic gravity field character, that is geoid height and spatial gravity anomaly. The global gravity field model WDM94 to degree and order 360

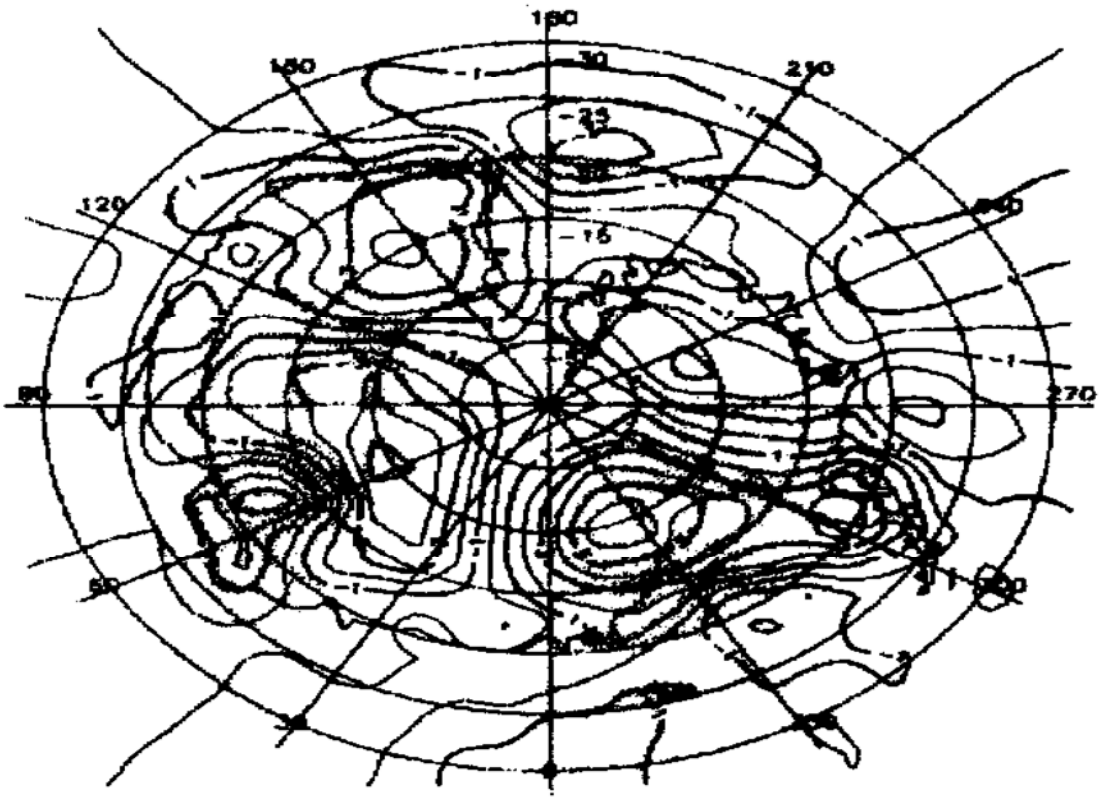

Fig. 5. The isolines of geoid differences between WDM94 model and JGMOSU model (unit: m, interval of isolines: 1 m).

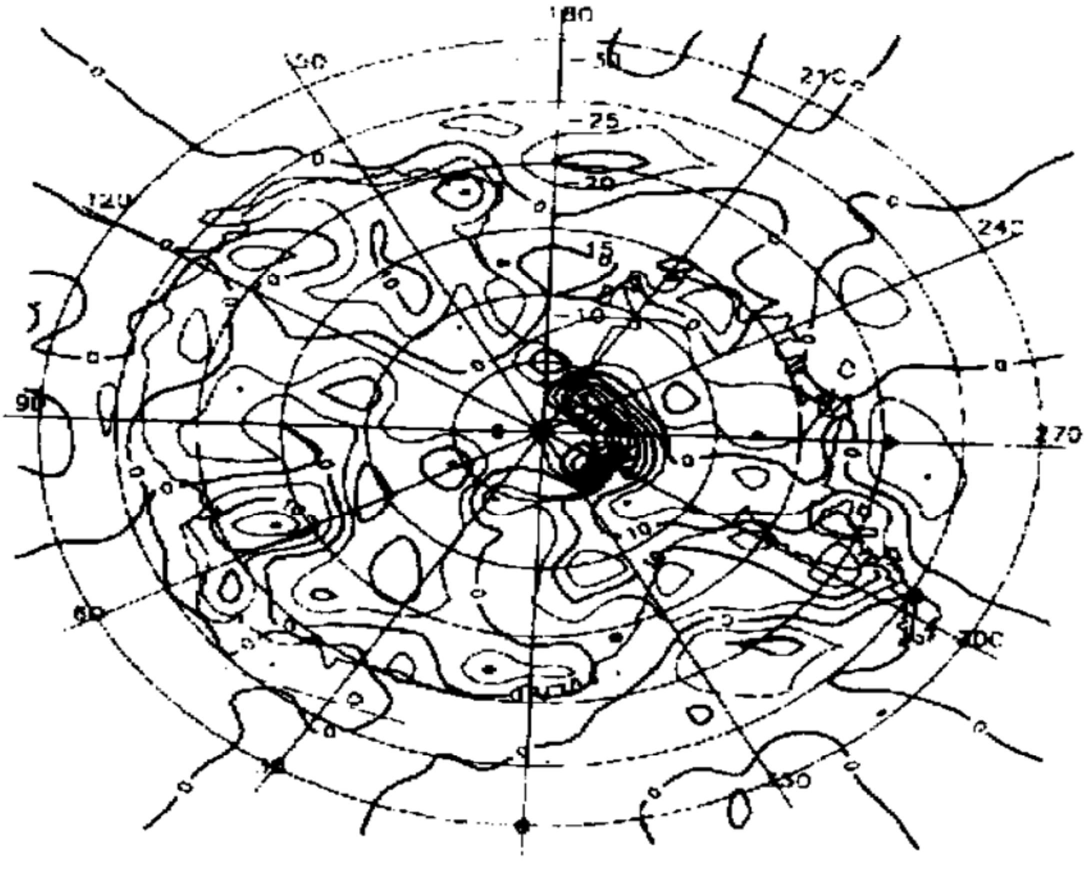

Fig. 6. The isolines of air gravity anomaly differences between WDM94 model and JGMOSU model(unit: mGal, interval of isolines: 5 mGal).

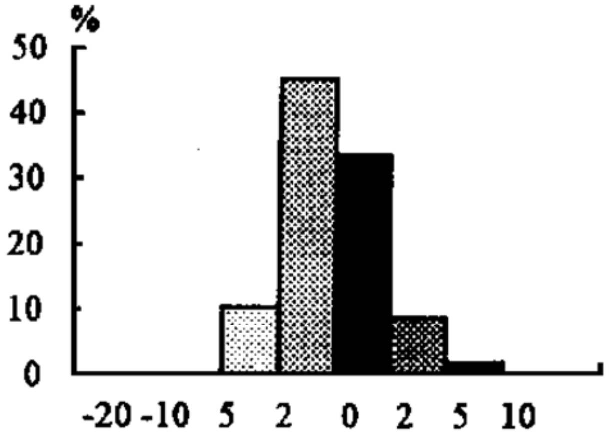

Fig. 7. The histogram of geoid differences between WDM94 model and OSU91 model (X axis unit: m).

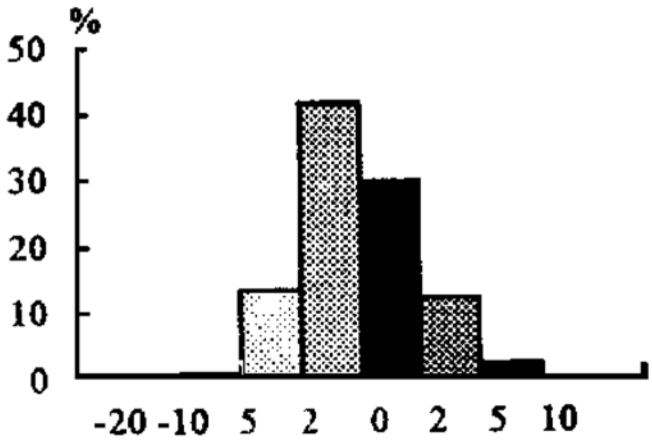

Fig. 8. The histogram of geoid differences between WDM94 model and JGMOSU model (X axis unit: m)

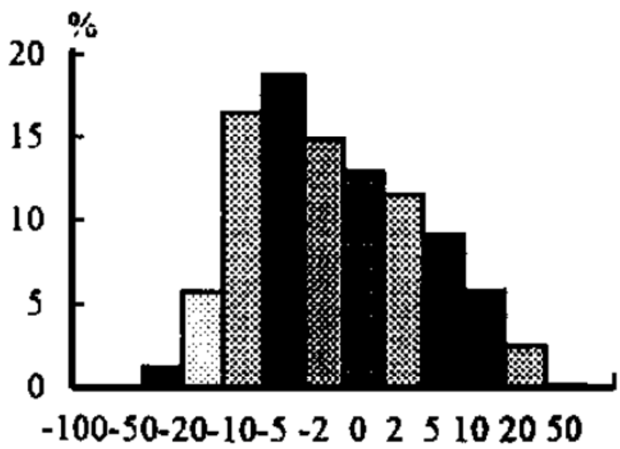

Fig. 9. The histogram of geoid differences between WDM94 model and OSU91 model (X axis unit: mGal).

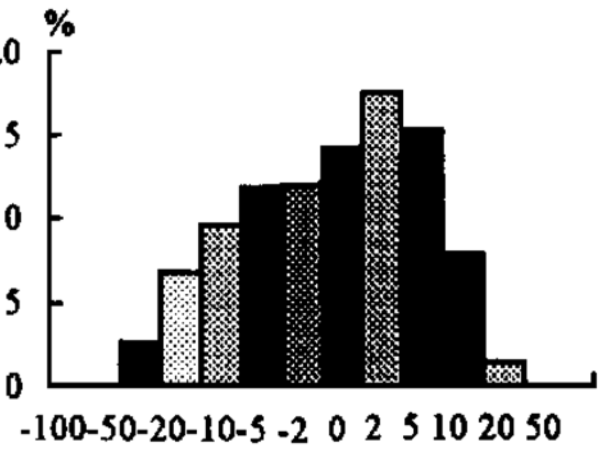

Fig. 10. The histogram of geoid differences between WDM94 model and JGMOSU model (X axis unit: mGal).

and OSU91, JGMOSU were used to calculate the $30' \times 30'$ geoid and air gravity anomaly of Antarctic circle (range: $-60° - -90°$), then these data and results were compared and analyzed respectively. The results have great and important value for the research on the character and distribution of Antarctic inner-Earth matter, land ice-snow load and crust structure, glacier melting, viscoelastic Earth and it´s geophysical interpretation.

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