

# Digital filter technology and its application to geomagnetic pulsations in Antarctica

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**Abstract** Digital filter technology is an important method in study of geomagnetic pulsations in Antarctica. The signals received by pulsation magnetometer on the ground include various types of magnetic pulsations. Some types of pulsations or some frequency bands of pulsations can be extracted from the signals by means of digital filter technology because types of pulsations are defined according to their frequency range. In this paper usual digital filter technology is provided for study of magnetic pulsations in Antarctica and some examples are introduced.

**Key words** Antarctica, magnetic pulsation, filter.

## 1 Introduction

Geomagnetic pulsations are quick variations of geomagnetic field and a kind of ULF waves below 5.0 Hz observed on the ground. It has become an important tool to research the magnetosphere and ionosphere because the ULF waves in magnetospheric space and disturbance of space current system are main excitation sources in space. Antarctica is a region where interaction between the solar wind and the geomagnetic field is very active and geomagnetic pulsations are abundant. Geomagnetic pulsations are related to aurora, magnetic storm, magnetospheric substorm, and ionospheric storm.

Geomagnetic pulsations are received on the ground by means of various magnetometers including flux gate magnetometer and induction magnetometer. These magnetometers have high sensitivity, broad frequency band, and A/D digital acquisition system that could receive all electromagnetic signals in frequency range of pulsations. Therefore, digital signal processing is needed because various kinds of geomagnetic pulsations are overlapped. Digital filter technology is very important to separate various geomagnetic pulsations in processing.

Digital filter technology has been developing in digital signal processing and provides many methods of digital filter. We give some feasible digital filter technology used in the investigation of Antarctic magnetic pulsations.

## 2 Choice condition of digital filter for magnetic pulsation

Digital filters fall into two classes: classical filter and modern filter (Zhong and Hu 1988). Principle of the classical filter is that useful and removed components of input sig-

nal  $x(n)$  have different frequency bands. When input signals pass through the classical filter, the components of undesired frequency band are removed effectively. As concerns the function of classical filter, there are low-pass (LP), high-pass (HP), band-pass (BP), and band-reject (BR) filters. According to data with noise, modern filter can estimate theoretically the characteristics of signal or signal itself by the use of a set of optimal estimation options derived from statistical feature of random signals. For example, Wiener filter, Kalman filter, adaptive filter etc. belong in this class of the filter.

Pulsation magnetometer uses super-low-drift amplifier so that the rate of signal to noise is very high, and there is not spectrum interval overlapping of signal and noise. Therefore, classical filter is satisfactory in study of magnetic pulsations. In this paper we reply the problem how to use classical filters in data analyses of magnetic pulsation.

In respect to operation method, classical filters can be divided into infinite impulse response digital filter (IIR) and finite impulse response digital filter (FIR). These two types of filters have great differences either in function or in design method. FIR filter can be designed directly according to given frequency features, while IIR filter can be designed by use of mature analogous filters, such as Butterworth filter, Chebyshev filter, ellipse filter etc..

In respect to performance, attenuation properties of band-pass and band-reject can reflect the quality of filters. FIR filter is not easy to get good attenuation properties just like IIR filter. So samples of temporal series need to be long enough for FIR to get better attenuation properties. But FIR filter has prominent virtues: system is stable and reliable, linear phase can be realized easily, and many band-pass (or band-reject) can be designed. Later two properties can not be realized for IIR filter.

Observation of magnetic pulsations at Zhongshan Station is for a long time and is continuous, so that the length of temporal series can be chosen arbitrarily. As long as a data segment is long enough, the defect of FIR filter could be overcome. The virtue that can realize linear phase is necessary to study magnetic pulsations. For example, when you study propagation properties of magnetic pulsations, you must determine phase difference between two stations in order to get propagation direction and velocity. If phase difference of magnetic pulsations passing through digital filter deforms nonlinearly, the conclusion will be wrong. Therefore, FIR digital filter is mainly applied to study of magnetic pulsations in Antarctica.

### 3 Design of digital filter

Design of FIR digital filter is based on some approximation of frequency properties for ideal filter. There are approximation methods, such as window function, frequency sampling, best consistent approximation, etc..

Fig. 1 gives principle of classical digital filter that is a linear and stable system. Input and output are indicated in Fig. 1. There are two methods to express the input and the output (Bath 1974). The expression in frequency domain is written as follows:

$$G(\omega) = F(\omega)H(\omega) \quad (1)$$

where  $F(\omega)$ ,  $G(\omega)$ ,  $H(\omega)$  indicate the input, output, and the transition function, respectively. Another is the expression in temporal domain written as follows:

$$g(t) = f(t) * h(t) \quad (2)$$

where  $f(t)$ ,  $g(t)$ ,  $h(t)$  indicate the input, output, and impulse response, respectively. Character \* indicates the composition.

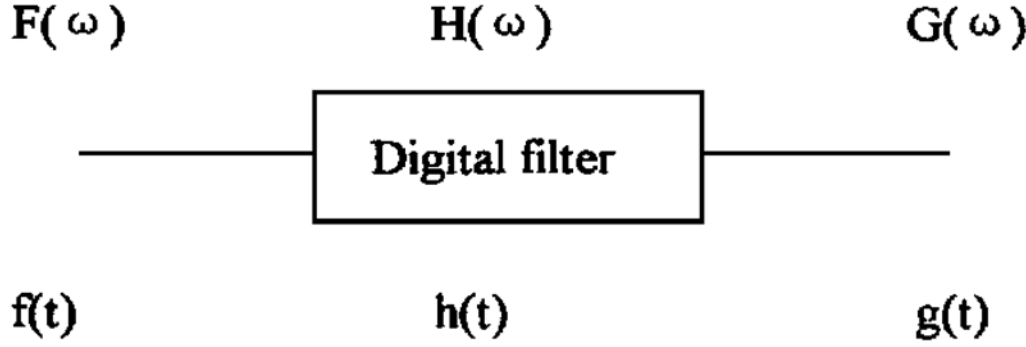


Fig. 1. The scheme of digital filter.

Considering the transition function of an ideal low-pass filter,  $H_d(e^{j\omega})$ , its amplitude  $H_d(e^{j\omega})$  equals to 1 when  $\omega \leq \omega_c$ , and equals to zero when  $\omega > \omega_c$ , its phase  $\Phi(\omega) = 0$ , so that the response of unit sample response is written as follows:

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\omega}) e^{j\omega n} d\omega = \frac{1}{2\pi} \int_{-\omega_c}^{\omega_c} e^{j\omega n} d\omega = \frac{\sin(\omega_c n)}{\pi n} \quad (3)$$

Obviously,  $h_d(n)$  is an infinite symmetric series with respect to  $h_d(0)$ . Such a digital filter can not be realized in physics because of its infinity and non-causality. If  $h_d(n)$  is cut off and replaced just like  $h_d(-M/2), \dots, h_d(0), \dots, h_d(M/2)$ ,  $h(n)$  can be written as follows:

$$h(n) = h_d(n - \frac{M}{2}) \quad \text{where } n = 0, 1, 2, \dots, M \quad (4)$$

Thus  $h(n)$  has the causality and finite length that is  $M+1$ . The transition function of designed digital filter can be written as follows:

$$H(z) = \sum_{n=0}^M h(n) z^{-n} \quad (5)$$

The frequency response of  $H(z)$  is approximately  $H_d(e^{j\omega})$  and has linear phase. If phase frequency response is defined as  $\Phi(\omega) = -(M\omega/2)$ , according to principle of linear phase unit sample response can be written as follows:

$$h_d(n) = \frac{1}{2\pi} \int_{-\omega_c}^{\omega_c} e^{-j\frac{M\omega}{2}} e^{+j\omega n} d\omega = \frac{\sin((n - \frac{M}{2})\omega_c)}{\pi(n - \frac{M}{2})} \quad (6)$$

$h_d(n)$  becomes a finite series symmetric with respect to  $n = M/2$  by means of original cut-off method,  $n = 0, 1, 2, \dots, M$ . Propounding  $h(n) = h_d(n)$ ,  $H(z)$  can be constructed with (5), but phase of the output in temporal domain moves up  $M/2$  samples. This problem can be solved in programming of computer.

A great problem of FIR filter is Gibbs' phenomenon. As  $M$  increases, the ripples appear in passed band. And these ripples can not disappear with increase of  $M$ , but the great upper impulse is near high threshold. It stands to reason that infinite  $h_d(n)$  is cut off into a finite series  $0-M$  equivalent to putting a rectangular window which length is  $(M+1)$  on the infinite  $h_d(n)$ . The rectangular window generates big boundary values that form Gibbs phenomenon in the composition with transition function.

In order to decrease Gibbs' phenomenon the window function with little boundary values must be used. Window function method for designing FIR filter chooses better window function to cut  $h_d(n)$  off at the cost of broadening transition belt. We have chosen effective Hamming window in filter design of magnetic pulsations after many tests. The digital filters of HP and BP have been designed by use of same principle.

In fact, a set of finite unit sample responses is designed to approximate frequency properties of desired ideal filter in window function method of FIR digital filter. Fourier series is applied in this method to approximate the frequency properties of ideal filter. Other methods are very similar to this. For example, frequency sampling method is a kind of interpolation method that is ensured for approximation of digital filter and ideal filter at sample point. Best consistent approximation method uses principle of Chebyshev's best consistent approximation and desires to approximate transition function of ideal filter. These methods can not be introduced in detail here.

When we deal with data of magnetic pulsations, all data in a day can be divided into some 20 min segments that have 600 points with sampling rate of 2 s. Firstly, the waveforms of H and D components and Fourier's spectrum of H component must be got on the screen of computer. Then each segment must pass through digital filter which functions and window types are chosen according to waveform and spectrum. Finally, filtered waveform, spectrum, and polarization properties or propagation properties of the segment are displayed on the screen of computer.

#### 4 Application of digital filter technology to magnetic pulsations in Antarctica

Digital filter technology is very important in digital signal processing of magnetic pulsations in Antarctica. For example, digital filters were adopted in investigation of polarization properties for various magnetic pulsation (Yang and Liu 1999) and in study of propagation properties between two and more stations (Liu *et al.* 1999) because some types of magnetic pulsations have to be chosen from mixed received signals. Some examples such as the polarization properties of Pc3 pulsations at Zhongshan Station of Antarctica and the propagation characteristics of Pc3 pulsations between Zhongshan and Davis stations are introduced as follows.

Fig. 2 gives an example in study of polarization characteristics of Pc3 pulsations at Zhongshan Station. Fig. 2(a) gives the observation of H and D components and the Fourier's spectrum of H component at Zhongshan Station from 1500 LT to 1520 LT on February 24, 1996 before filtering. Obviously, the observation would include many types of magnetic pulsations. It is seen from the spectrum analysis that there are two types of magnetic pulsations in H component: narrow-band Pc5 with peak of 1.67 mHz and broadband Pc3 with main frequency of 28.37 mHz. In order to study polarization properties H and D components in the segment are filtered with respect to the main frequency with band of  $\pm 1.7$  mHz for band-pass. Fig. 2(b) gives waveforms of filtered H and D components, variation of ellipticity and azimuth with local time, spectrum of filtered H component, and polarization ellipses for first half and latter half of the segment. It is seen from Fig. 2(b) that the main frequency is 28.37 mHz, the maximum amplitude of H component is 0.86 nT/s, the average ellipticity is  $-0.32$ , and the average azimuth of major axes is  $-40.94^\circ$  in this segment. By means of this method the polarization proper-

ties of some pulsation event can be studied, and statistical regularities are investigated for diurnal, monthly, annual variations and for various types of magnetic pulsations.

Fig. 3 gives an example in study of the propagation properties of Pc3 pulsations between Zhongshan and Davis Stations. Fig. 3(a) gives the observations of D component at two stations, the corresponding phase differences of various frequencies, and the Fourier spectrum of D component at Zhongshan Station from 0900 LT to 0920 LT on August 3, 1996 before filtering. The phase difference is got by means of cross spectrum analysis.

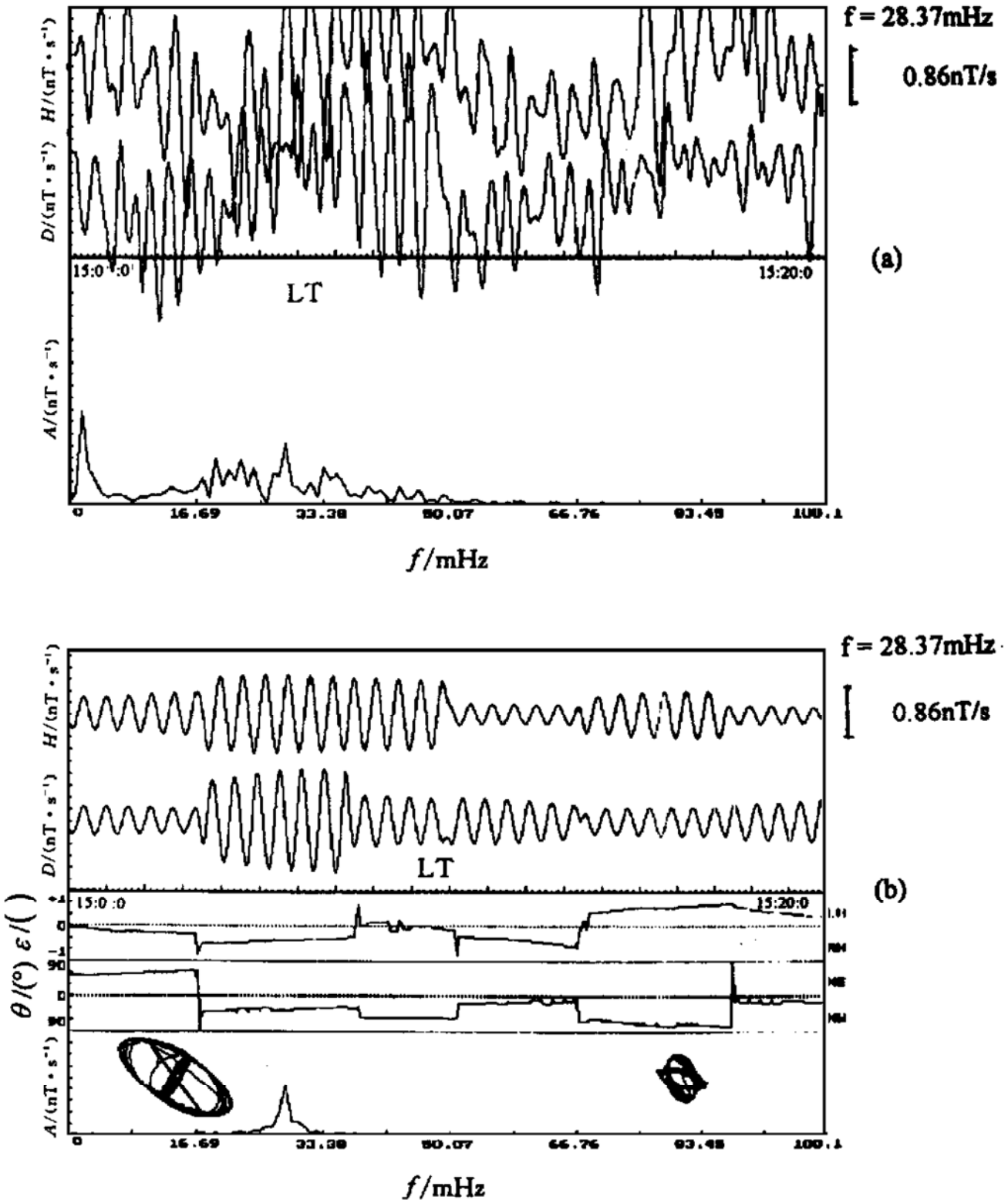


Fig. 2. The results of polarization analysis for Pc3 pulsations at Zhongshan Station on February 24, 1996.

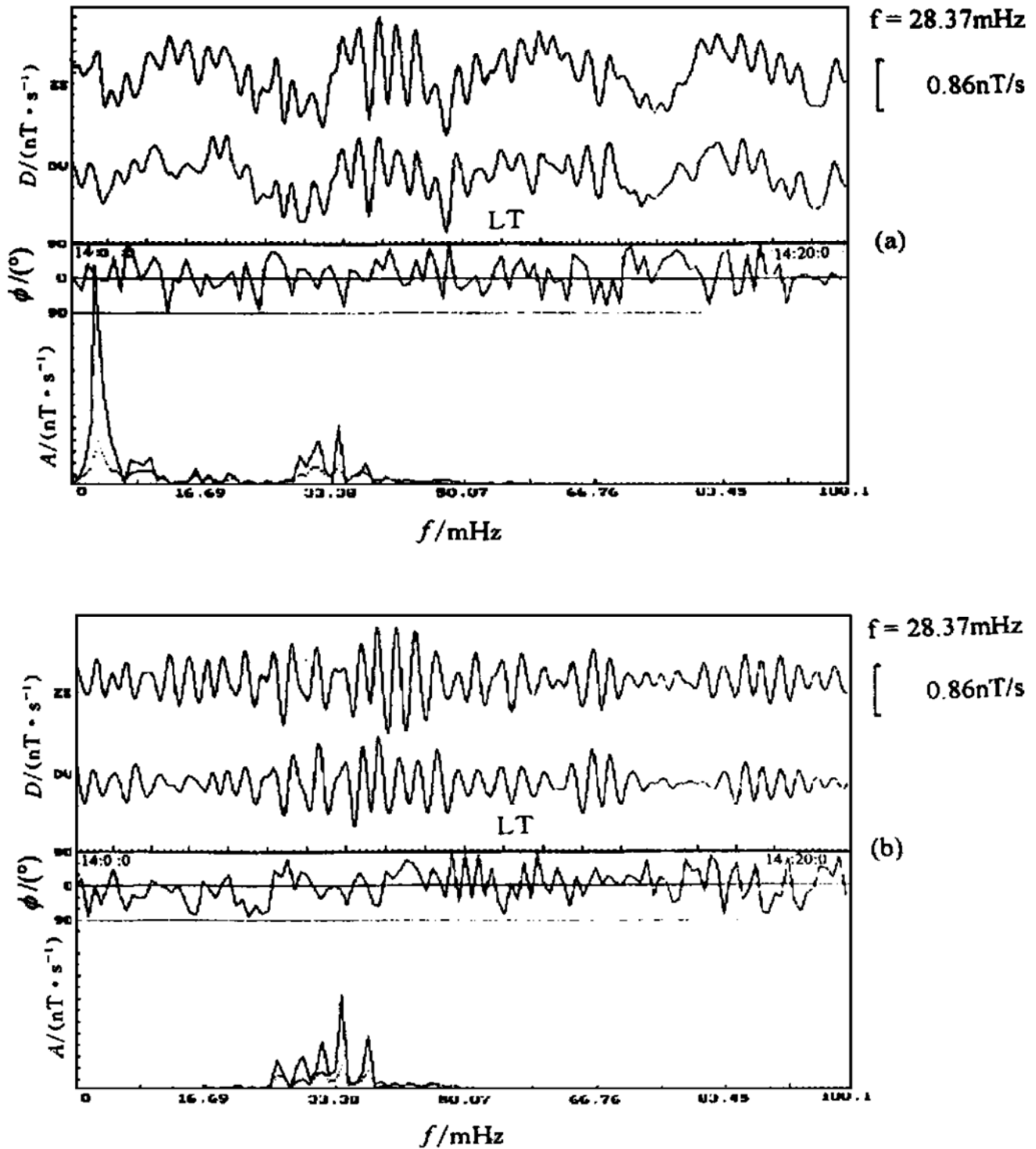


Fig. 3. The results of propagation analysis for Pc3 pulsations at Zhongshan Station and Davis Station on August 3, 1996.

Obviously, the observations would include many types of magnetic pulsations. It is seen from spectrum analysis that there are three types of magnetic pulsations in D component: the narrow-band Pc5 with prominent peak of 3.34 mHz, the Pc4 pulsations with low amplitudes and the broadband Pc3 with main frequency of 34.22 mHz. In order to study propagation properties of Pc3 pulsation between two stations, data in same segment are filtered from 22.46 mHz to 50.07 mHz for band-pass. Fig. 3(b) gives waveforms of filtered D components at two stations, variation of phase differences with frequencies, spectrum of filtered D component at Zhongshan Station. We always study the phase dif-

ference and wave number of main frequency for filtered Pc3 pulsations. It is seen from Fig. 3(b) that the main frequency is still 34.22 mHz, the phase difference of main frequency is  $-6.16^\circ$ , and the wave number is  $-4.57$ , eastward propagation is dominant in this segment. By means of this method the propagation properties of some pulsation event can be studied, and statistical propagation regularities of diurnal variation, monthly variation, annual variation for various types of magnetic pulsations are investigated.

## 5 Conclusion

We have applied digital filter technology to study of magnetic pulsations in Antarctica and have got good results in the investigation of polarization characteristics and propagation properties. Higher requirement of digital filter technology will be set with deep development of magnetic pulsations in Antarctica. It needs us to study digital filter technology deeply and search new filter technology as well as to improve the quality of filter in order to satisfy the demand of investigation for magnetic pulsations in Antarctica.

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