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Snow water content estimation from measured snow temperature

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Abstract The vertical temperature profiles of snow and sea ice have been measured in the Arctic during the 2nd Chinese National Arctic Research Expedition in 2003 (CHINARE2003). The high-resolution temperature profile in snow is solved by one-dimensional heat transfer equation. The effective heat diffusivity, internal heat sources are identified. The internal heat source refers to the penetrated solar radiation which usually warms the lower part of the snow layer in summer. By temperature gradient analysis, the zero level can be clarified quantitatively as the boundary of the dry and wet snow. According to the in situ time series of vertical temperature profile, the time series of water content in snow is obtained based on an evaluation method of snow water content associated with the snow and ice physical parameters. The relationship of snow water content and snow temperature and temporal-spatial distribution of snow water content are presented.

Key words Arctic, sea ice, snow, water content

1 Introduction

Sea ice in Polar Regions plays an important role in the global climate system. It affects the atmospheric and oceanic circulations via mutual interactions[1, 2]. Consequently it shows impacts on climate and hydrological development in China[3, 4]. There is usually a snow cover on top of ice, introducing various impacts on sea ice thermodynamics (i.e. insulating effect, snow to ice transformation). For climate research, sea ice thermodynamic model normally employs snow and ice, respectively[5]. Furthermore, snow, especially in summer, can affect the variability of ice parameters extracted from the remote sensing image[6], because melting or freezing of snow with the change of air temperature results in the change of the water content in snow. The water content in snow determines the physical, thermal and photoelectric properties of the whole snow layer. Therefore, the determination of the water content in snow is very important for the study of its role in the thermal process of sea ice and the obtainment of sea ice parameters from the satellite remote sensing. The measurement with dielectric moisture meter can be only limited in the single location and perturb the initial snow layer simultaneously. Although the change of the water content on space can be obtained, that on time can not be obtained[7]. However, it is well known that the measurement of snow / ice temperature field is much easier to be carried out[8]. In this paper, based on the measured temperature profiles during the 2nd Chinese National Arctic Research Expedition (CHINARE2003), one-dimensional heat transfer equation is employed to identify the effective heat diffusivities and the internal heat source that is used to determine the snow layer with water and that without water. According to the physical parameters of dry snow and water, the approach to evaluate the water content in snow is introduced. The time-space distribution of water content shows the snow in the state of melting or freezing in the vertical profile of snow depth. If the snow keeps in freezing and cold, there is no water; if the snow keeps in melting, it is warmer and there is water inside. Corresponding one measurement, the vertical profile of water content in snow just gives the state of water contents and positions in snow. From the time series of measurement and calculation of water content, the melting water immigration in snow can be evaluated. Because the density of water is higher than that of snow, the water immigrates downward to ice surface. The snow surface usually has not water due to the water immigration downward and freezing in cold air. The position with water in snow is deeper in cold air and shallower in warmer air. The water content is higher near the interface between snow and ice. Therefore, combined with the time series of snow temperature, the water immigration in snow and the snow melting or freezing can be shown. The water content distributions and the measured ice and snow thickness[9] can be used in further sea ice thermodynamic research.

2 One-dimensional thermodynamic model

In order to obtain the water content in snow layer, arbitrarily two snow layers on top of sea ice are defined. Usually, it can be expected that phase transformations (snow-melting) occurred in these two layers. The heat transfer equation including internal heating source item has been applied for each snow layer:

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(\alpha_{eff}(T)_1 \frac{\partial T}{\partial x}\right) + f_1(t, x, T), \; x \in \Omega_1 \qquad (1)$$

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(\alpha_{eff}(T)_2 \frac{\partial T}{\partial x}\right) + f_2(t, x, T), \; x \in \Omega_2 \qquad (2)$$

where $T = T(t, x)$ is temperature; t is time confined within a domain I, $t \in I = [0, t_e]$, $t_e \in R^+$ being the final time; (The) Ω_1 and Ω_2 are the vertical 1-D spatial upper and lower domains, respectively; $\alpha_{eff}(T)_i = (k_{eff})_i / [(C_{eff})_i \cdot (\rho_{eff})_i]$, $i = 1, 2$, are the effective upper and lower heat diffusivities; $(k_{eff})_i$, $(C_{eff})_i$ and $(\rho_{eff})_i$ are the effective heat conductivity, effective specific heat and effective density of each layer; $f_i(t, x, T)$ is the heat source term, which can be positive, negative or zero. Because heat diffusivity is a monotonously decreasing function of temperature[10], and temperature varies within a relatively small range in the Arctic summer, it can be assumed that the effective heat diffusivity of snow is linear with respect of temperature, i.e. $\alpha_{eff}(T)_i = a_i + b_i T$ $(i = 1, 2)$, where a_i and b_i are constants. The items $f_i(t, x, T)$ $(i = 1, 2)$ depend on time, space and temperature and therefore are assumed to be control parameters and to be obtained in identification procedure.

based on the measured temperature field.

Equation (1) and (2) are discretized by using the half implicit difference scheme[11]. Based on the measured vertical profiles of snow temperature during CHINARE2003, the numerical simulation can be performed in the upper and lower layers according to the approach of Schwartz alternating direction, then the corresponding heat diffusivities can be obtained. The detailed procedure is as follows. The distances of the nodes corresponding to space and time are $\Delta x = 0.002$ m and $\Delta t = 30$ s, respectively. The measured temperature data at $x = 0.035$ m and $x = 0.415$ m are simulated to serve as the upper and lower boundaries of the whole snow layer. The measured temperature data at 0:00 on 2003.8.24 are simulated to be the initial condition of the snow layer. The snow layer can be divided into two layers at any measured $x' \in [0.035\,\text{m}, 0.415\,\text{m}]$, and the measured temperature data at x' are simulated as the lower boundary of the upper snow layer. The parameter identification procedure can be carried out between $x = 0.035$ m and x'. The maximum x' subject to $f_1 = 0$ can be defined as the boundary between the dry and wet snow layers. In this paper, one can get $x' = 0.315$ m. According to the initial condition of the whole snow layer and the boundary condition of the dry and wet snow layers, the heat diffusivities of the two layers can be obtained by parameter identification and numerical simulation with the measured temperature data at $x = 0.265$ m and $x = 0.365$ m as the identification data of the dry and wet snow layers, respectively. The effective heat diffusivity (m^2/s) of the dry snow layer is

$$\alpha_{eff}(T)_1 = (6.0 - 3.0 \times 10^{-7} T) \times 10^{-7} \quad (3)$$

and that of the wet snow layer is

$$\alpha_{eff}(T)_2 = (7.2 - 1.4 \times 10^{-2} T) \times 10^{-8} \quad (4)$$

The corresponding heat sources (K/s) are $f_1 = 0$ and $f_2 = 1.13 \times 10^{-4}$, respectively. Figure 1 shows the time series of the measured and the modeled temperature data at $x = 0.265$ m and $x = 0.365$ m from 2003.8.24 0:00 to 2003.8.27 0:00. The error analysis shows that the relative error between the measured temperature and the modeled result is less than ±0.2%.

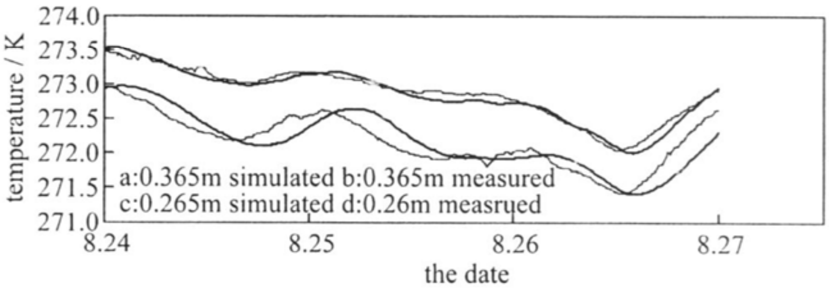

Fig. 1 The measured and simulated snow temperature at 0.265 m and 0.365 m (August 24-27, 2003).

3 The inversion of the water content in snow

The snow layer can be regarded as the mixture of dry snow, air and water. Since the wet snow layer is composed of water, snow crystal and air, and the dry snow layer is composed of snow crystal and air, the wet snow layer can be assumed to be composed of dry snow and water. The physical and thermodynamic properties of the wet snow layer are the composition of those of dry snow and water. The water content in the snow layer is inversed based on the determination of the thermal parameters by the measurable snow-layer temperature field. The approach has been applied successfully in the mixed materials of solid and liquid[12].

For a micro volume unit which includes a measured temperature $T(x, t)$ (x denotes the space coordinate of the measured location, t denotes the time coordinate, T denotes the temperature of the measured location), if the water content is assumed to distribute uniformly in the unit, and the percent of water content is denoted by θ, the dry snow content can be written by $1-\theta$. If the density of dry snow is denoted by ρ_s and the density of water is denoted by ρ_w, the effective density of the wet snow can be written by

$$\rho_{eff} = \rho_s \rho_w / [(\rho_s - \rho_w) \cdot \theta + \rho_w] \tag{5}$$

where $\rho_s = k_s / ((\alpha_{eff}(T))_1 \cdot C_s)$, k_s and C_s denote the heat conductivity and the specific heat of dry snow, respectively. k_s and C_s are dependent on the density of dry snow and temperature[13,14], i.e.

$$k_s = 2.845 \times 10^{-6} \rho_s^2 + 2.7 \times 10^{-4} \times 2^{(T-233)/5} \tag{6}$$

$$C_s = (92.88 + 7.364T) \times 10^3 \tag{7}$$

Since the temperature of the Arctic ice layer in summer varies around the freezing point, the density of the pure water can be given as $\rho_w = 999.2$ kg/m^3. According to the result of Nazintsev (1963)[15], the effective specific heat of the wet snow layer can be described by

$$C_{eff} = C_w \theta + C_s(1-\theta) + L d\theta/dT, \tag{8}$$

where C_w denotes the specific heat of the pure water.

If the mixture of dry snow and water is assumed to be a saturate system, the effective heat conductivity of the wet snow can be obtained by Ling and Zhang[16].

$$k_{eff} = k_w^{\Theta} \cdot k_s^{1-\Theta} \tag{9}$$

where $\Theta = \rho_{eff} \cdot \theta / \rho_w$ is the volume proportion of water, k_w is the heat conductivity of water. The latent heat L, the heat conductivity k_w and the specific heat of water C_w are given[17] as

$$L = \rho_w [333.2 + 4.995(T - 273.15) + 0.02987(T - 273.15)^2], \tag{10}$$

$$k_w = 0.11455 + 1.6318 \times 10^{-3} T, \tag{11}$$

$$C_w = 4.20843 + 0.111362(T - 273.15) + 5.12142 \times 10^{-3}(T - 273.15)^2 + 9.3482 \times 10^{-5}(T - 273.15)^3 \tag{12}$$

Substituting equation (3)-(5) and (7)-(10) to (6), equation (6) can be simply expressed as

$$d\theta/dT = g(\theta, T) \tag{13}$$

where g is a continuous function on θ and T.

Based on the measured temperature, it is assumed that the water content is zero at the highest temperature of the lower boundary of dry snow layer, that is, $\theta(272.99\text{ K}) = 0$ at $x = 0.315$ m. Therefore, equation (11) can be solved numerically in $[T_0, T_m] = [272.99\text{ K}, 273.99\text{ K}]$, where $T_m = 273.99$ K is the highest temperature in the wet snow layer. According to numerical simulations, the estimated relationship between the water content and temperature is shown in Figure 2.

The figure shows that the water content in snow increases with the increasing temperature. Because the highest temperature in wet snow layer is only 0.84 K greater than the freezing point, and the lowest temperature corresponding to the nonzero water content is only 0.16 K less than the freezing point, such variation of temperature is reasonable in nature. In particular, during the Arctic summer, the penetrated solar radiation is strong enough to cause the internal melting of snow and further to increase the temperature of melted snow and slush beyond the freezing point.

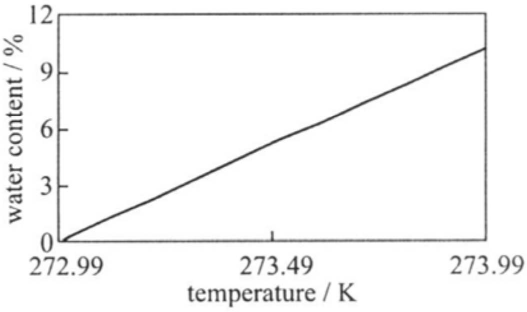

Fig. 2 Estimated snow water content versus snow temperature

4 The analysis of water content in the snow layer

Snow temperature field has spatial and temporal variations and so does the snow water content. For a given time t, the vertical profile of temperature in snow is a function of depth x, therefore the water content θ is also a function of x denoted by $\theta(x)$. Analogously, the effective density of snow corresponding to the profile is also regarded as the function of x, denoted by $\rho_{eff}(x)$. For a given spatial segment in snow layer, the water content can be regarded as the integration of that in each unit, therefore, at a given time t, in the given space range, the water content θ can be given by

$$\theta^{*} = \int_{x_1}^{x_2} \theta(x)\, \rho_{eff}(x)\, dx \Big/ \int_{x_1}^{x_2} \rho_{eff}(x)\, dx \qquad (14)$$

where x_1 and x_2 are the depths corresponding to the upper and lower boundaries of the given spatial segment, respectively.

The upper and lower positions of the given segment are determined by the minimal thickness which ensures the simulation to be carried out reasonably. According to the temperature field and snow pits measured during CHINARE2003, from 0.035 m, the location of the first temperature probe under the snow, to 0.415 m, the surface of sea ice, the snow layer is divided into 200 layers with a interval of 0.002 m. The temperature corresponding to every micro snow layer at the time t can be obtained by simulating the measured temperature on the coordinate x. Because the water content θ in snow is a function of temperature, and the effective density of snow is a function on the water content and temperature, the water content of every micro snow layer can be obtained by simulation. Combining with the water content of every micro snow layer, the vertical profile of water content in snow layer at the time t can be calculated. Combining with the vertical profiles at different time, the time-space distribution of water content can be obtained.

Figure 3 shows the time-space distribution of water content from 2003.8.24 0:00 to 2003.8.27 0:00 with the time interval of 600 s and the micro snow layer thickness of 0.002 m. Since no water exist from the surface of the snow layer to the depth of 0.002 m by analysis, figure 3 only shows the time-space distribution of water content from 0.200 m to 0.500 m. The water content in snow is zero at 0.415 m, namely sea ice surface. Compared with figure 1, the snow melts and the water content increases when the temperature in snow increases with the increase of the external air temperature. On the contrary, the water in snow freezes and the water content decreases even arrives at zero. From space, at given *t*, the water content increases with the increasing depth and tends to zero at the ice surface. During the whole measured period, the maximum water content is 10.21% which occurs at the depth of 0.403 m, and the minimum is zero. The effective density (kg/m^3) of snow varies in the range of 369.72-405.30, which shows that there is phase transformation in the snow ice layer in summer. The absorption and release of the heat in phase transformation influence the melting speed and play an adjusting role.

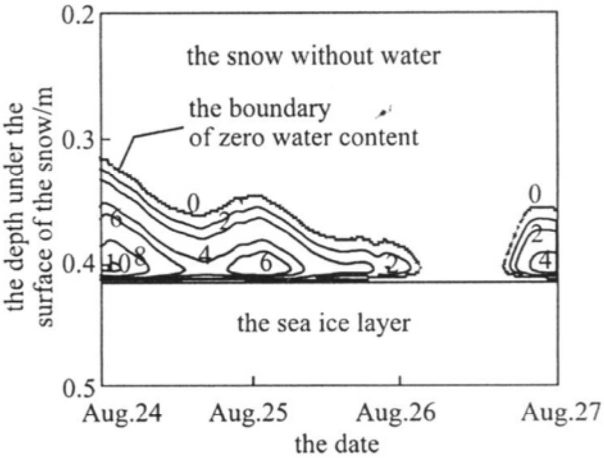

Fig. 3 Time-space distribution of water in snow.

5 Conclusions

(1) Based on one-dimensional heat transfer equation and the measured temperature in CHINARE2003, the snow layer is regarded as the mixture of dry snow and water, and the effective diffusivities changing with the depth and the heat source are identified. The snow is considered as wet snow or dry snow corresponding to whether there is phase transformation or not, in terms of whether the heat source is nonzero or not, respectively. It provides an approach by which the water content can be inversed based on the known physical and thermodynamic parameters of dry snow and water.

(2) Based on the calculated micro snow layers, the vertical profile of water content corresponding to the temperature profile at given time can be obtained. According to the vertical profiles of water content corresponding the different time, the time-space distribution of water content can be obtained.

(3) It is an indirect approach to inverse the water content in snow layer based on the measured temperature. The approach provides a simple and effective way to indirectly observe the water content in snow layer. The accurate of this method will be done with the measured water content and temperature in snow in future.

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References

[1] Bian LG, Gao ZQ, Lu LH *et al.* (2003): Observational estimation of heat budgets on drifting ice and open water over the Arctic Ocean. Science in China, Ser. D, 33(2): 139-147.

[2] Zhao JP, Gao GP, Jiao YT (2004): The water warming in the middle and deeper layer of the Chukchi sea plateau and its adjacent area in 1999. Science in China, Ser. D, 34(2): 188-194.

[3] Chen JN, Chu JT, Xun LY (2004): ENSO events associated with the variation of the Antarctic sea ice extent. Advance in Water Science, 15(1): 56-60.

[4] Li HP, Xu YM, Rao SQ (2000): Analysis on influence of sea ice in north pole area on runoff in the upper Yellow River during flood season. Advance in Water Science, 11(3): 284-290.

[5] Winton M (2000): A reformulated three-layer sea ice model. Journal of Atmospheric and Oceanic Technology, 17: 25-31.

[6] Belchansky G and Douglas DV (2002): Seasonal comparisons of sea ice concentration estimates derived from SSM/I, OKEAN, and RADARSAT data. Remote Sensing of Environment, 81: 67-81.

[7] Zhou SQ, Nakawo M, Hashimoto S *et al.* (2004): Field measurements of the irreducible water saturation in snow. Journal of Hydraulic Engineering, 2: 29-33.

[8] Li ZJ, Zhang ZH, Dong XL, *et al.* (2004): The new technique of the key indices in the growth of the Arctic sea ice. Advance in Natural Science, 14(9): 1077-1080.

[9] Li ZJ, Han M, Qin JM *et al.* (2005): The advance of the automonitoring and detecting the thickness change of sea ice. Advance in Water Science, 16(5): 753-757.

[10] Yen YC (1981): Review of thermal properties of snow, ice and sea ice, CRREL Report 81–10, U. S. Army Cold Regions Research and Engineering Laboratory, Hanover, New Hampshire.

[11] Bai YL, Zhao HQ, Zhang X *et al.* (2005): The model of heat transfer of the arctic snow-ice layer in summer and numerical simulation. Journal of Industrial and Management Optimization, 1(3): 405-414.

[12] Shoshany Y, Prialnik D, Morris PM (2002): Monte Carlo modeling of the thermal conductivity of porous cometary ice. Icarus, 157(1): 219-227.

[13] Ebert EE, Curry JA (1993): An intermediate one-dimensional thermodynamic sea ice model for investigating ice-atmosphere interaction. Journal of Geophysical Research, 98: 10085-10109.

[14] Anderson EA (1976): Point energy and mass balance model of a snow cover. NOAA Technical Report, NWS 19, National Oceanic and Atmosphere Administration, Washington, DC.

[15] Nazintsev YL (1963): On the role of thermal processes in the melting of ice and transformation of relief of multi-year fields in the Central Arctic. Problemy Arktiki i Antarktiki, 12: 29-76.

[16] Ling F, Zhang TJ (2004): A numerical model for surface energy balance and thermal regime of the active layer and permafrost containing unfrozen water. Cold Regions Science and Technology, 38: 1-15.

[17] Osterkamp TE (1987): Freezing and thawing of soils and permafrost containing unfrozen water or brine. Water Resources Research, 23(12): 2279-2285.