Chinese Journal of Polar Science, Vol. 19, No. 2, 237 – 248, December 2008

Mechanical and numerical models for sea ice dynamics on small-meso scale

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Received November 17, 2008

Abstract On small-meso scale, the sea ice dynamic characteristics are quite different from that on large scale. To model the sea ice dynamics on small-meso scale, a new elastic-viscous-plastic (EVP) constitutive model and a hybrid Lagrangian- Eulerian (HLE) numerical method are developed based on continuum theory. While a modified discrete element model (DEM) is introduced to model the ice cover at discrete state. With the EVP constitutive model, the numerical simulation for ice ridging in an idealized rectangular basin is carried out and the results are comparable with the analytical solution of jam theory. Adopting the HLE numerical model, the sea ice dynamic process is simulated in a vortex wind field. The furthering application of DEM is discussed in details for modeling the discrete distribution of sea ice. With this study, the mechanical and numerical models for sea ice dynamics can be improved with high precision and computational efficiency.

Key words sea ice dynamics, constitutive model, numerical method, discrete element model, small-meso scale.

1 Introduction

The ice cover plays an important role for global climate in the polar region, and also brings a series of engineering problems in the sub-polar. To solve the sea ice problems on various scales, the numerical simulations of sea ice dynamics have been investigated for more than several decades. To improve the computational precision and efficiency, some efforts have been carried out from two aspects, i. e. developing reasonable constitutive models and effective numerical methods.

The existing constitutive models for sea ice dynamics mainly includes viscous plastic (VP) model[1], elastic plastic (EP) model[2], elastic-viscous -plastic model coupled with granular flow dynamics[3,4], and anisotropic models[5-7]. The VP model has been applied most widely in the polar areas on large temporal-spatial scale, and in the Baltic Sea and Bohai Sea on meso-scale[8-10]. Various modifications on VP model have been attempted either for computational efficiency or for physical process [11-15]. However, the VP model can not model the elastic deformation of sea ice on meso-small scale, even it works well in polar regions on large scale.

In the present study, an elastic-viscous-plastic (EVP) model for sea ice dynamics is developed. In this model, the elastic-viscous behavior under small strain and strain rate, the plastic rehology under large strain, the Mohr-Coulomb yielding criterion, and the hydrostatic pressure are considered. Similar to the VP and EP models, the EVP model also assumes the ice as a two-dimensional isotropic continuum. To check the validity of EVP model, the ice ridging process in a rectangular basin is simulated.

In the numerical simulations of sea ice dynamics, a series of numerical models have been developed in Eulerian, Lagrangian Coordinates or their combinations since 1970s. The Eulerian Finite Difference Method (FDM) is the most widely applied numerical approach in the sea ice dynamics[9,10,16]. Meanwhile, some Lagrangian methods, such as Smoothed Particle Hydrodynamics (SPH), were developed to overcome the numerical diffusion in advection term of FDM[17-20]. But in the applications of Lagrangian method, the neighbor particle searching costs a huge computing time, which limits its applications in the long term sea ice simulation. Recently, some coupled Lagrangian-Eulerian methods, such as Particle-in-cell (PIC) approach, were developed and applied in various cold regions[13,21,22].

To improve the precision and computational efficiency of coupled Lagrangian-Eulerian method, a hybrid-Lagrangian-Eulerian (HLE) model is developed with adopting the SPH concept. In this HLE model, the ice motion, thickness and concentration are calculated with Lagrangian ice particles, and the ice velocity is simulated in Eulerian grids. Between the Lagrangisn particles and Eulearian grids, the Gaussian integral function is adopted to transfer the ice variables. With this HLE model, the ice drifting process in a vortex wind field is modeled.

In the field observations and satellite images, it is shown that the sea ice performs as granular materials[19,23]. The ice floes have a large size range, which can be more than 100 km on large scale, or less than 1m on small scale[5,19,24]. To model the dynamic characteristics of sea ice cover in discrete state, the discrete element model (DEM) has been developed[18,25,26,27]. In conventional DEM, the ice cover is described as rigid blocks with constant size and thickness, but the parameters of ice blocks can not be determined exactly. Moreover, the huge computational cost of DEM is the key limitation. Therefore, a modified DEM is introduced for sea ice dynamics in the present study. In this modified DEM, the most novel portion is the particle plastic deformation, which can model the ice rafting and ridging, and affect the thickness of ice particles.

This paper is organized as follows. Firstly, the EVP constitutive law is established for sea ice dynamics, and is validated with the ice riding in a rectangular basin. Then, the HEL numerical method is introduced and applied to model the ice dynamic process in a vortex wind field. Finally, the modified DEM is discussed in details to simulate the sea ice dynamics under various ice conditions.

2 Elastic-Viscous-Plastic (EVP) Constitutive Model for Sea Ice Dynamics

On meso-small scale, the ice cover appears rafting, ridging and breaking up randomly. Considering the sea ice dynamics characteristics described above, an EVP model is established. With this EVP model, the ice ridging process in an idealized rectangular basin is simulated.

2.1 *EVP constitutive model for sea ice dynamics*

Similar to many constitutive models of sea ice dynamics, the present model includes an elastic- viscous equation, a Mohr-Coulomb yield criterion and an associated normality plastic flow rule. In addition, the hydrostatic pressure depending on the ice thickness and concentration is also considered. The Elastic-viscous-plastic model is depicted in Fig. 1. In this figure, σ is the normal stress, ε_e and ε_p are the elastic and plastic strain, respectively. The spring, dashpot, and sliding blocks represent the elastic, viscous and plastic properties of sea ice, respectively.

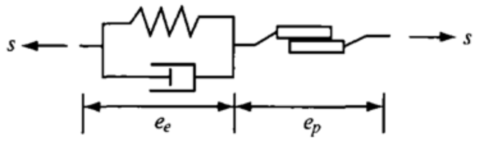


Fig. 1 Elastic-viscous-plastic model for sea ice dynamics. Here, σ is the normal stress, ε_e and ε_p are the elastic and plastic strain, respectively. This figure mainly includes three portions, i. e. , the elastic spring, the viscous dashpot and the plastic sliding block.

Considering the hydrostatic pressure, the stress-strain (strain rate) relationship of elastic-viscous equation is given as

$$\sigma_{ij} = 2\eta_V\dot{\varepsilon}_{ij} + (\zeta_V - \eta_V)\dot{\varepsilon}_{kk}\delta_{ij} + 2G\varepsilon_{ij} + (K - G)\varepsilon_{kk}\delta_{ij} - P_r\delta_{ij} \tag{1}$$

where σ_{ij} and ε_{ij} are the stress and strain tensors; $\dot{\varepsilon}_{ij}$ is the strain rate; ζ_v and η_v are the bulk and shear viscosities; δ_{ij} is the Kronecker delta symbol, and $\delta_{ij} = 1$ $(i = j)$ or 0 $(i \neq j)$. P_r is the hydrostatic pressure. K and G are the bulk and shear elastic modulus with $K = E/[2(1-v)]$ and $G = E/[2(1+v)]$. Here E is the Young's modulus, and v is the Poisson's ratio. The elastic modulus of ice cover is affected by ice concentration with $E = E_0(N/N_{max})^m$, here E_0 is the Youns's modulus of sea ice at maximum concentration; In this study, we set $E_0 = 1.0 \times 10^5$ Pa; m is the empirical constants, normally $m = 15$; N is the ice concentration, and N_{max} is its maximum value[11][28].

For the plastic yielding criteria of sea ice, the Mohr-Coulomb friction law has been introduced into ice dynamics[6]. Generally, the Mohr-Coulomb yield surface is a hexagonal cone in 3-D principle stress space. For the sea ice dynamics, the principle stress in the z direction is its mean stress, which is set as $\sigma_3 = -P_0$. Thus, a hexagonal curve can be truncated from the hexagonal cone. The yielding function is determined by the three parameters, namely frictional angle, cohesion and hydrostatic pressure. The Mohr-Coulomb yield curve is constructed with the three lines, i. e. shear, compressive and tensile surfaces (as shown in Fig. 2), and can be written as

$$\sigma_1 = K_D\sigma_2 + 2c\sqrt{K_D} \tag{2}$$

$$\sigma_C = -K_CP_0 - 2c\sqrt{K_C} \tag{3}$$

$$\sigma_T = -K_DP_0 + 2c\sqrt{K_D} \tag{4}$$

where σ_1 and σ_2 are the principle stresses; σ_C and σ_T are the compression and tension strengths; P_0 is the mean pressure in the vertical direction of ice cover; c is the cohesion of sea ice; The parameters K_D and K_C can be determined with $K_D = \tan^2(\pi/4 - \varphi/2)$ and $K_C = \tan^2(\pi/4 + \varphi/2)$. Here ϕ is the internal friction angle of sea ice, and is shown in Fig. 2.

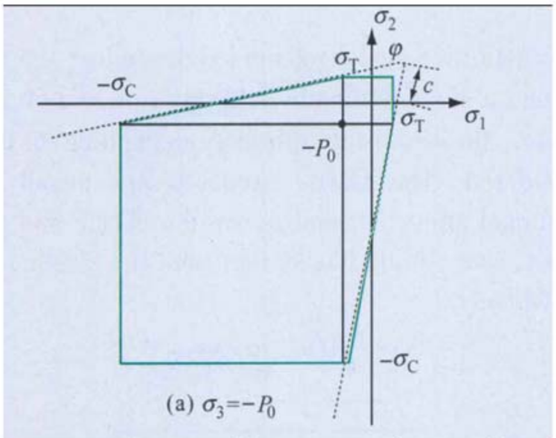


Fig. 2 Mohr-Coulomb yielding criterion of EVP model. Here σ_1 and σ_2 are the principle stresses; σ_C and σ_T are the compression and tension strength; P_0 is the mean pressure in the vertical direction of ice cover; c is the cohesion of sea ice.

Considering the influence of ice concentration, the mean vertical hydrostatic pressure can be expressed as[11]

$$P_0 = \left(1 - \frac{\rho_i}{\rho_w}\right)\frac{\rho_i g t_i}{2}\left(\frac{N}{N_{max}}\right)^j \tag{5}$$

where ρ_i and ρ_w are the densities of ice and water; g is the gravity; t_i is the ice thickness; The horizontal hydrostatic pressure can be calculated as $P_r = K_0 P_0$. Where P_r is the horizontal hydrostatic pressure; K_0 is the transfer coefficient. In broken ice field without cohesion, there is the expression $K_0 = 1 - \sin\varphi$.

When the ice stress steps into the plastic state under large deformation, the principle stress will be on the yield curve. The total deformation includes two portions: elastic strain and plastic strain. When the stress is in the elastic state, the plastic strain is zero. While in the plastic state, the plastic strain can be determined with Drucker's Postulate. Here, the associated normal flow rule is adopted. In this way, the Mohr-Coulomb yielding function is used as the plastic potential function, and the direction of plastic strain rate is normal to the yielding curve.

2.2 *Numerical simulation of ice ridging with EVP model*

A rectangular basin with length L and width B is covered by uniform layer of ice with thickness t_{i0} and concentration N_{i0}. Under constant wind and current drags, the sea ice piles up at downstream. The internal ice resistance increases with the ice thickness to balance the wind and current drag forces. Considering the bank friction, the steady ice ridge thickness profile can be obtained from the classical static ice jam theory [28]. The stress in x direction is calculated with the plastic limit analytical theory. In this ice ridging case, the ice volume remains constant, and the ridging length can be determined according to the thickness profile function.

The ice ridging process in a rectangular basin is simulated with EVP model. The model parameters in the simulation are listed in Table 1. Considering bank friction, the mean thickness and its contour are plotted in Fig. 3 and Fig. 4. Under the given wind and current condition, the thickness profile approaches steady state after 4 hours. The ice thickness near the bank is thinner than that along the center line of the channel. The width-averaged ice thickness is consistent with the analytical solution. The shear and normal stresses in x direction are presented in Fig. 5. It can be seen the shear stress is higher at bank boundary, and approaches zero at the middle basin, and is symmetric with the centerline. The normal stress in x direction is proportion to the ice thickness.

Table 1. Parameters used in the ice ridging simulation

Parameter	Definition	Value	Parameter	Definition	Value
B	Width of ice field (m)	500	L	Initial ice length (m)	4500
t_{i0}	Initial ice thickness (m)	0.2	N_0	Initial ice concentration (%)	100
V_a	Wind speed (m/s)	8.0	V_w	Current speed (m/s)	0.1
C_a	Wind drag coefficient	0.015	C_w	Current drag coefficient	0.02

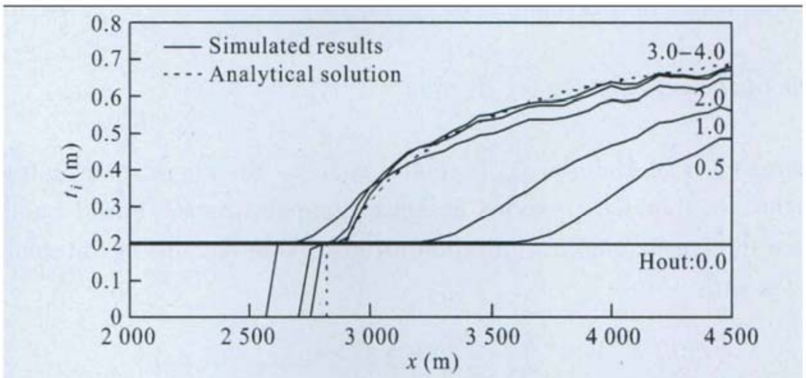


Fig. 3 Width-averaged ice thickness simulated and analytical solution considering bank friction.

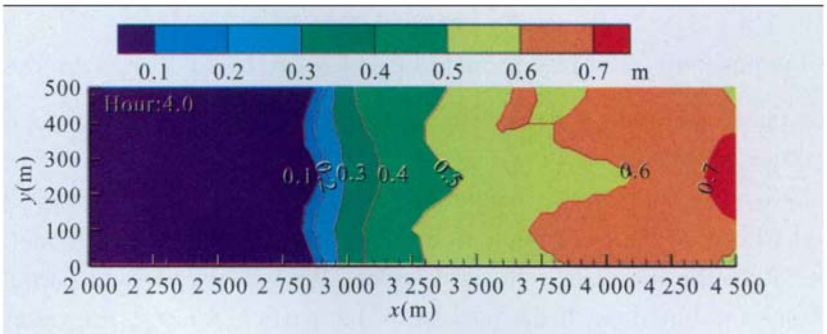


Fig. 4 Ice thickness contour simulated with EVP model.

Therefore, the ice ridging process in rectangle basin can be simulated validly with this EVP constitutive model. The simulated ice thickness profile at steady state is agreeable with the analytical solutions of classical ice jam theory.

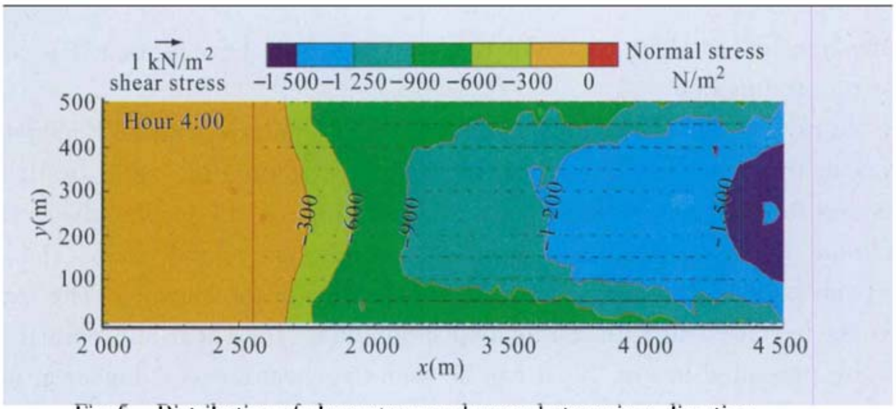


Fig. 5 Distribution of shear stress and normal stress in x direction.

3 Hybrid Lagrangian-Eulerian (HLE) Numerical Model for Sea Ice Dynamics

In this HLE numerical model, the sea ice velocity is simulated in Eulerian coordinates, and the ice thickness and concentration are determined in Lagrangian Coordinates. The Gaussian interpolating function links the sea ice variables between Eulerian and Lagrangian Coordinates. To examine the reliability of this HLE model, the sea ice dynamical drifting in a vortex wind filed is simulated.

3.1 HLE numerical model for sea ice dynamics

The ice parameters of Eulerian grids note, such as thickness, concentration, can be determined by its neighbor ice particles at Lagrangian locations. Based on the SPH concept, the sea ice thickness and concentration on grid node $\boldsymbol{X}_{ij}$ can be interpolated from its neighbor particles with

$$\tilde{h}_{\mathrm{i}}(\boldsymbol{X}_{ij}) = \sum_{k=1}^{N}\left[\frac{m_k}{M_k}W(\boldsymbol{X}_{ij} - \boldsymbol{r}_k, h_0)h_{\mathrm{i}}(\boldsymbol{r}_k)\right] \tag{6}$$

$$\tilde{N}(\boldsymbol{X}_{ij}) = \sum_{k=1}^{N}\left[\frac{m_k}{M_k}W(\boldsymbol{X}_{ij} - \boldsymbol{r}_k, h_0)N(\boldsymbol{r}_k)\right] \tag{7}$$

where $\boldsymbol{X}_{ij}$ is the position vector of gird node (i,j); $h_{\mathrm{i}}(\boldsymbol{r}_k)$ and $N(\boldsymbol{r}_k)$ are the thickness and concentration of sea ice particle k; $\tilde{h}_{\mathrm{i}}(\boldsymbol{X}_{ij})$ and $\tilde{N}(\boldsymbol{X}_{ij})$ are the estimations of ice thickness and concentration on grid node (i,j).

When the ice thickness and concentration at grid nodes are determined, their velocities can be solved with the momentum equation using finite differential method (FDM) at Eulearian grids. With the ice velocity at grid nodes, the velocity of each particle can be estimated with Gaussian function. If the position of ice particle k is $\boldsymbol{r}_k$, the velocity of this particle can be interpolated from its grid neighbors with

$$\tilde{V}(\boldsymbol{r}_k) = \sum_{i}\sum_{j}\left[\frac{m_{ij}}{M_{ij}}W(\boldsymbol{X}_{ij} - \boldsymbol{r}_k, h_0)\boldsymbol{V}_{i,j}\right] \tag{8}$$

where $\tilde{V}(\boldsymbol{r}_k)$ is the evolution of velocity vector of ice particle k; $\boldsymbol{V}_{ij}$ is the velocity vector of grid (i,j); m_{ij} and M_{ij} are the mass and mass density of grid (i,j), respectively.

When the velocity of particle k at time step t^n is determined with Eq. (8), its position vector at time step t^{n+1} can be calculated with

$$\boldsymbol{r}_k(t^{n+1}) = \boldsymbol{r}_k(t^n) + \Delta t\tilde{V}(\boldsymbol{r}_k(t^n)) \tag{9}$$

where Δt is the time step.

In this HEL model, the mass density of each ice particle is used to determine the thickness and concentration of ice particles. Thus, the numerical diffusion in solving continuity equation in Eulerian Coordinates is avoided.

Considering the ice particle k at position $\boldsymbol{r}_k$, its mass density can be estimated with its neighbor particles by[25][18]

$$\tilde{M}(\boldsymbol{r}_k) = \sum_{j=1}^{N} m_j W(\boldsymbol{r}_k - \boldsymbol{r}_j, h_0) \tag{10}$$

The smooth length of ice particles h_0 can be adjusted with its current mass density. Considering the density mass of ice particle k with $\tilde{M}(\boldsymbol{r}_k) = \rho_i N(\boldsymbol{r}_k) h_i(\boldsymbol{r}_k)$, the concentration of this particle is given by

$$N(\boldsymbol{r}_k) = \frac{\tilde{M}(\boldsymbol{r}_k)}{\rho_i h_i(\boldsymbol{r}_k)} \tag{11}$$

If $N(\boldsymbol{r}_k) > 1.0$, the ice ridging occurs, and $N(\boldsymbol{r}_k) = N_{max} = 1.0$ is tenable. The thickness of ice particle k can be calculated with $h_i(\boldsymbol{r}_k) = \tilde{M}(\boldsymbol{r}_k)/(\rho_i N_{max})$.

Based on the description above, the ice velocity, concentration and thickness are all determined in Lagrangian and Eulerian Coordinates, and the numerical simulation of sea ice dynamics can be performed with the computational loops above.

3.2 Numerical simulation of ice drifting in a vortex wind field

The numerical test of sea ice drifting in a vortex wind field was constructed by Flato[13] to estimate the PIC approach for sea ice dynamics. Here, this vortex wind field is also adopted to verify the HLE model. In this vortex wind field test, the upper half of 500 km × 500 km rectangular domain is covered by uniform ice cover with thickness of 1.0m and concentration of 1.0. The lower half is open water. The vortex wind field is defined with[13]

$$\boldsymbol{W}(r) = \min\left\{\omega r, \frac{\lambda}{r}\right\}\boldsymbol{k} \times \frac{\boldsymbol{r}}{r} \tag{12}$$

where $\boldsymbol{W}$ is the wind vector, r is the distance to the vortex center, $\boldsymbol{r}$ is the position vector to the vortex center, and the vortex center position is (250 km, 200 km). Here $\omega = 0.5 \times 10^{-3}\ \mathrm{s}^{-1}$ and $\lambda = 8 \times 10^5\ \mathrm{m}^2/\mathrm{s}$. The sea ice dynamical porcess in this vortex wind field is simulated in 10 days with HLE model. In this simulation, the time step is 180 s, grid size is 10 km × 10 km, and the initial ice particle size is 5 km × 5 km. The simulated ice particles distribution and the mean ice thickness on 5th and 10th day are plotted in Fig. 6. It can be found that the ice dynamcis can be well simulated with this HLE model. Especailly, the ice edge has a sharp shape, which shows the high precision of this model.

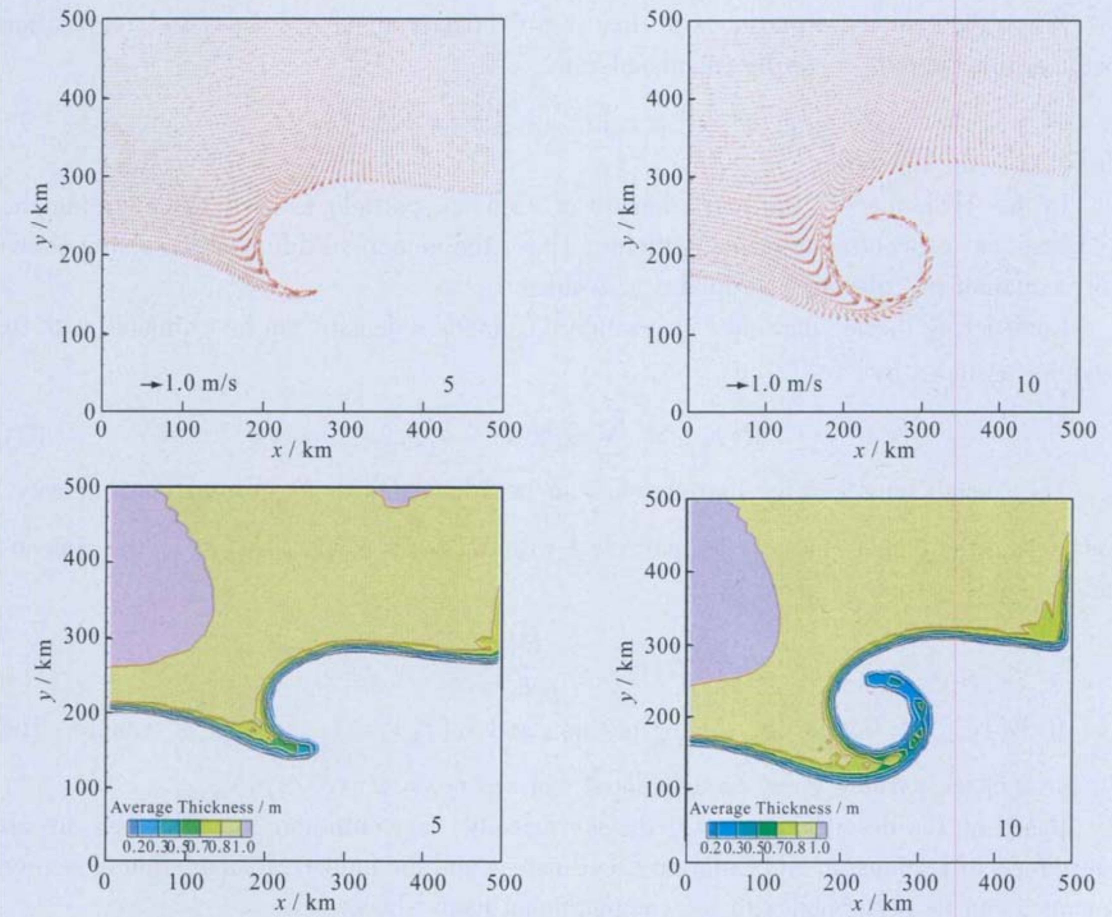


Fig. 6 Velocity vector and mean ice thickness simulated with HLE method.

4 Modified Discrete Element Model (DEM) for Sea Ice Dynamics

In natural conditions, the sea ice appears as granular materials on meso-small scale, which is quite different from that on large scale. As shown in Fig. 7, either the sea ice satellite image on meso scale or the sea ice picture taken on small scale in the Liaodong Bay, the sea ice performs as discrete media. The similar observations are also described in other studies[19][23]. To model the dynamic characteristics of sea ice in discrete state, the discrete element model (DEM) has been developed on various scales[3][29][4][27][30].

Even for the continuum theory of sea ice dynamics, some constitutive models were also developed based on the granular flow dynamics[3][29][31]. But most researchers would like simulate the sea ice dynamics with DEM directly. Lepparanta *et al.* (1990)[32] and Shen *et al.* (2004)[26] established the DEM to consider the sea ice dynamics at broken-up ice field on small scale. Hopkins (1996, 1999)[4][30] simulated the ice ridging/rafting on mesoscale. For the case of large scale, the DEM was also applied in the polar region recently[27]. With the previous studies above, it can be found that the sea ice dynamics can be simulated with DEM on various scales.

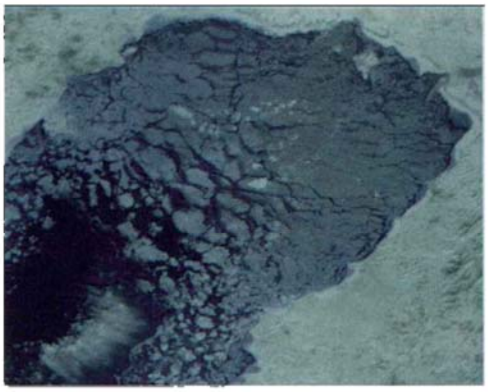
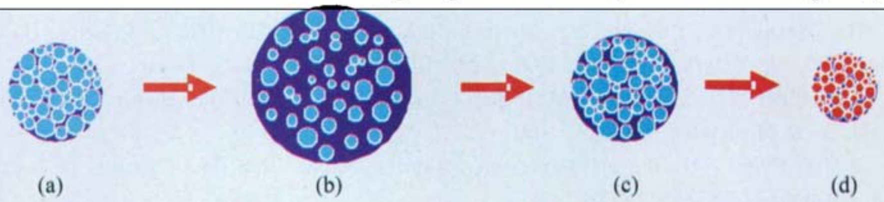


Fig. 7 Satellite image of sea ice on meso scale (left) and the sea ice picture on small scale (right) in the Liaodong Bay. The satellite image was obtained with MODIS remote data on January 15, 2001. The sea ice picture was taken on January 10, 2000 in the JZ20-2 oil field of the Liaodong Bay.

In the conventional DEM for sea ice dynamics, the ice cover is described as rigid blocks with constant size and thickness. In the case of meso-large scales, the parameters of ice blocks (i. g. ice thickness, position, velocity, size and shape, etc.) can not be determined exactly. Moreover, the huge computational cost of DEM is the key limitation. It is impossible to describe all of the ice blocks and their broken, rafting or ridging processes. Therefore, a modified DEM is considered for sea ice dynamics in the present study. This modified DEM adopt the concept of SPH approach, and one ice particle is an assembly of small ice blocks. Therefore, the ice particle is not a real ice block, and has its statistical information depending on its ice floes inside.

The interaction among ice particles is determined with the elastic-viscous-plastic contact model. The contact force model mainly consists of four portions: damping force proportional to velocity, elastic force based on stiffness and overlap distance, Mohr-Coulomb friction force and the plastic force based on the soil mechanics. The inter-particle contact force model can be linear or nonlinear. In this modified DEM, the most novel portion is the plastic deformation, which can consider the ice rafting and ridging. Since one particle in the DEM is constructed as an assembly of ice floes, the particle size can be adjusted based on the interactions with its neighbors, while its concentration and mean thickness can also be changeable accordingly (as shown in Fig. 8). The sea ice floes have an initial dense packing in a sea ice package with high concentration (as shown in Fig. 8(a)). Under the wind and current actions, the ice cover can be packed in loose or dense conditions (as shown in Fig. 8(b)-(c)). In the dynamic process of ice particle size, the total mass of the ice particle is constant. When the concentration approaches its maximum value 1.0, the mean thickness will increase with the decreasing of particle size (as shown in Fig. 8(d)).



(a) (b) (c) (d)

Fig. 8 Changing process of sea ice particle in the modified DEM.

A Mohr-Coulomb yielding limit is imposed on the particles to represent ice ridging. Any stress on the particle, exceeding the yielding criterion, results in particle deformation. The yielding criterion in the model is only dependent on the compressive stress and can be determined with Eq. (3). The yielding stress σ_C is also given in Fig. 2 above. In this modified DEM, cohesion forces can be neglected by setting $c = 0$. Since tensile stress only occurs when ice particles cohere together, tensile stress is also ignored normally. If considering the thermodynamics and adhesion among ice particles, the refreezing and broken-up of ice cover can be described exactly.

5 Conclusions

In the numerical simulation of sea ice dynamics on meso-small scale, it is a key problem to establish mechanical and numerical models with high precision and computational efficiency. In this study, an EVP constitutive model and a HLE numerical method are presented to model the sea ice dynamics on meso-small scales. Meanwhile, a modified DEM is also introduced to model the discontinuous state of ice cover. The EVP constitutive model and the HLE numerical method are validated with the ice riding in a uniform wind field and with ice dynamic process in a vortex wind field. Different from the continuum concept, the modified DEM is based on the discrete media mechanics, and can be applied well in the numerical simulation of sea ice dynamics on meso-small scale.

Based on the studies here, the several works will be considered further. The one is to combine the EVP constitutive model and the HLE numerical method for sea ice dynamics to improve the simulation precision and computational cost simultaneously. The other one is to optimize the numerical algorithm of the modified DEM of sea ice dynamics under various ice conditions. Finally, the thermodynamics should be considered to coupling with sea ice dynamics in both of continuum model and discrete method.

Acknowledgements The helpful discussions with Profs. Hung Tao Shen and Hayley H Shen from Clarkson University of USA are appreciated. The study is supported by the National Natural Science Foundation of China (Grant No. 40206004) and partly by the open foundation of Key Laboratory of Polar Science of Science of the State Oceanic Administration, China (Grant No. KP2007004).

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